Диференціальні рівняння

__Завдання 25__. Розв'язати диференціальне рівняння.

25.1. $xy' = 1 + y^2$.

25.2. $yy'\sqrt{1+x^2} = x\sqrt{1+y^2}$.

25.3. $y' = \dfrac{x^2 y + y}{\sqrt{4+y^2}}$.

25.4. $x + xy + y'(y + xy) = 0$, .

25.5. $\left(y - x^2 y\right) y' = 4x - 5xy^2$.

25.6. $y' = \dfrac{y}{\sqrt{x^2+1}}$.

25.7. $y' \operatorname{tg} x = y$.

25.8. $\left(e^{2x} + 5\right) y' = ye^{2x}$.

25.9. $e^{2x}\left(2y - 1\right) y' = y$.

25.10. $\left(x + 4\right) y' = y^2 - 1$.

25.11. $\left(1 + e^x\right) yy' = e^x$.

25.12. $\sqrt{4 - x^2}\, y' + xy^2 + x = 0$.

25.13. $\left(e^x + 8\right) y' = ye^x$.

25.14. $. 2x + 2xy^2 + \sqrt{2 - x^2}\, y' = 0$

25.15. $y' \operatorname{ctg} x = y^4$.

25.16. $y'y\sqrt{1 - x^2} = \sqrt{5 + y^2}$.

25.17. $\left(2x - xy^2\right) dx = \left(y + yx^2\right) dy$.

25.18. $y \ln y + xy' = 0$.

25.19. $xy' + y = y^2$.

25.20. $\sqrt{1 - x^2}\, y' + xy^2 + x = 0$.

25.21. $xy' - 2y = yx^3$.

25.22. $xy' = y\left(1 + \ln y\right)$.

25.23. $\left(3 + e^x\right) yy' = e^x$.

25.24. $\sqrt{3 + y^2} + \sqrt{1 - x^2}\, yy' = 0$.

25.25. $y' \sin x = y \ln y$.

25.26. $\left(1 + e^x\right) yy' = e^x$.

25.27. $yy' = e^x\left(4 + y^2\right)$.

25.28. $\sqrt{4 - x^2}\, y' = 3x + xy^2$.

25.29. $y' \operatorname{ctg} y = x^3$.

25.30. $y' = 2\sqrt{y} \ln x$.

__Завдання 26__. Знайти загальний розв'язок диференціального рівняння.

26.1. $y' = \dfrac{y^2}{x^2} + 4\dfrac{y}{x} + 2$.

26.2. $y' = \dfrac{y}{x} + \sin\dfrac{y}{x}$.

26.3. $y' = \dfrac{x + y}{x - y}$.

26.4. $y' = \dfrac{y^2}{x^2} - \dfrac{y}{x}$.

26.5. $y' = \dfrac{y^2}{x^2} + 5\dfrac{y}{x} + 8$.

26.6. $y' = \dfrac{y}{x} + \operatorname{tg}\dfrac{y}{x}$.

26.7. $xy' \cos\dfrac{y}{x} = y \cos\dfrac{y}{x} - x$.

26.8. $y' = \dfrac{x + 2y}{2x - y}$.

26.9. $y' = \dfrac{y^2}{x^2} + 8\dfrac{y}{x} + 4$.

26.10. $xy' = y\left(1 + \ln\dfrac{y}{x}\right)$.

26.11. $xy' + 2\sqrt{xy} = y$.

26.12. $xy' = \sqrt{2x^2 + y^2} + y$.

26.13. $y' = \dfrac{y^2}{x^2} + 3\dfrac{y}{x} + 5$.

26.14. $xy' - y = x \operatorname{tg}\dfrac{y}{x}$.

26.15. $xy' = xe^{-\frac{y}{x}} + y$.

26.16. $xy' = y + 2x\sin^2\frac{3y}{x}$.

26.17. $y' = \frac{y^2}{x^2} + 7\frac{y}{x} + 9$.

26.18. $y' = \frac{x^2 + xy + y^2}{x^2}$.

26.19. $xy' = \sqrt{x^2 - y^2} + y$.

26.20. $xy' - y = y\ln\frac{y}{x}$.

26.21. $y' = 2\frac{y^2}{x^2} + 5\frac{y}{x} + 1$.

26.22. $xy' = y + 2x\,\mathrm{tg}\frac{3y}{x}$.

26.23. $xy' = 3\sqrt{x^2 - y^2} + y$.

26.24. $xy' = y + x\cos^2\frac{y}{x}$.

26.25. $xy' = y + x\sin^2\frac{2y}{x}$.

26.26. $y' = \frac{y^2}{x^2} + 9\frac{y}{x} + 16$.

26.27. $y' = \frac{y^2}{x^2} + 3\frac{y}{x} + 1$.

26.28. $xy' = 4\sqrt{x^2 + y^2} + y$.

26.29. $y' = 3\cos^2\frac{2y}{x} + \frac{y}{x}$.

26.30. $y' = \frac{y^2}{x^2} + 5\frac{y}{x} + 4$.

Завдання 27. Знайти розв'язок диференціального рівняння, що задовольняє задану початкову умову.

27.1. $y' - \frac{y}{x} = x^2$, $\qquad\qquad y(1) = 0$.

27.2. $y' - y\,\mathrm{ctg}\,x = 2x\sin x$, $\qquad y\left(\frac{\pi}{2}\right) = 0$.

27.3. $y' + y\cos x = \frac{1}{2}\sin 2x$, $\qquad y(0) = 0$.

27.4. $y' + y\,\mathrm{tg}\,x = \cos^2 x$, $\qquad y\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

27.5. $y' - \frac{y}{x+2} = x^2 + 2x$, $\qquad y(-1) = \frac{3}{2}$.

27.6. $y' - \frac{x}{x+1}y = e^x(x+1)$, $\qquad y(0) = 1$.

27.7. $y' - \frac{y}{x} = x\sin x$, $\qquad y\left(\frac{\pi}{2}\right) = 1$.

27.8. $y' - \frac{2x-5}{x^2}y = 5$, $\qquad y(2) = 4$.

27.9. $y' + \frac{y}{2x} = x^2$, $\qquad y(1) = 1$.

27.10. $y' + \frac{2x}{1+x^2}y = \frac{2x^2}{1+x^2}$, $\qquad y(0) = \frac{2}{3}$.

27.11. $y' + \frac{y}{x} = \sin x$, $\qquad y(\pi) = \frac{1}{\pi}$.

27.12. $y' + \frac{y}{x} = \frac{x+1}{x}e^x$, $\qquad y(1) = e$.

27.13. $y' - \frac{y}{x} = -2\frac{\ln x}{x}$, $\qquad y(1) = 1$.

27.14. $y' + \dfrac{1-2x}{x^2}y = 1$, $\qquad\qquad\qquad y(1) = 1$.

27.15. $y' + \dfrac{2}{x}y = x^3$, $\qquad\qquad\qquad y(1) = -5/6$.

27.16. $y' - \dfrac{2xy}{1+x^2} = 1 + x^2$, $\qquad\qquad y(1) = 3$.

27.17. $y' + \dfrac{y}{x} = 3x$, $\qquad\qquad\qquad y(1) = 1$.

27.18. $y' - \dfrac{y}{x} = -\dfrac{12}{x^3}$, $\qquad\qquad\quad y(1) = 4$.

27.19. $y' + \dfrac{3y}{x} = \dfrac{2}{x^3}$, $\qquad\qquad\quad y(1) = 1$.

27.20. $y' + 2xy = -2x^3$, $\qquad\qquad\quad y(1) = e^{-1}$.

27.21. $y' + \dfrac{xy}{2(1-x^2)} = \dfrac{x}{2}$, $\qquad\qquad y(0) = \dfrac{2}{3}$.

27.22. $y' + xy = -x^3$, $\qquad\qquad\qquad y(0) = 3$.

27.23. $y' - \dfrac{2}{x+1}y = e^x(x+1)^2$, $\qquad y(0) = 1$.

27.24. $y' + 2xy = e^{-x^2}\sin x$, $\qquad\qquad y(0) = 1$.

27.25. $y' - \dfrac{2y}{x+1} = (x+1)^3$, $\qquad\qquad y(0) = \dfrac{1}{2}$.

27.26. $y' - y\cos x = -\sin 2x$, $\qquad\qquad y(0) = 3$.

27.27. $y' - 4xy = -4x^3$, $\qquad\qquad\quad y(0) = -\dfrac{1}{2}$.

27.28. $y' - \dfrac{y}{x} = -\dfrac{\ln x}{x}$, $\qquad\qquad\quad y(1) = 1$.

27.29. $y' - 3x^2 y = \dfrac{x^2(1+x^3)}{3}$, $\qquad\quad y(0) = 0$.

27.30. $y' - \dfrac{y}{x} = -\dfrac{2}{x^2}$, $\qquad\qquad\quad y(1) = 1$.

Завдання 28. Знайти розв'язок диференціального рівняння, що задовольняє задані початкові умови.

28.1. $y''' = \sin x$, $\qquad\qquad\quad y(0) = 1$, $y'\ 0\ = 0$ $y''(0) = 0$.

28.2. $y''' = \dfrac{1}{x}$, $\qquad\qquad\qquad y(1) = \dfrac{1}{4}$, $y'(1) = y''(1) = 0$.

28.3. $y'' = \dfrac{1}{\cos^2 x}$, $\qquad\qquad y(0) = 1$, $y'(0) = \dfrac{3}{5}$.

28.4. $y''' = \dfrac{6}{x^3}$, $\qquad\qquad\quad y(1) = 0$, $y'(1) = 5$, $y''(1) = 1$.

28.5. $y'' = 4\cos 2x$, $\qquad\qquad y(0) = 1$, $y'(0) = 3$.

28.6. $y'' = \dfrac{1}{1+x^2}$, $\qquad\qquad y(0) = 0$, $y'(0) = 0$.

28.7. $xy''' = 2$, $\qquad\qquad y(1) = \dfrac{1}{2}$, $y'(1) = y''(1) = 0$.

28.8. $y''' = e^{2x}$, $\qquad\qquad y(0) = \dfrac{9}{8}$, $y'(0) = \dfrac{1}{4}$, $y''(0) = -\dfrac{1}{2}$.

28.9. $y''' = \cos^2 x$, $\qquad\qquad y(0) = 1$, $y'(0) = -\dfrac{1}{8}$, $y''(0) = 0$.

28.10. $y'' = \dfrac{1}{\sqrt{1-x^2}}$, $\qquad\qquad y(0) = 2$, $y'(0) = 3$.

28.11. $y'' = \dfrac{1}{\sin^2 2x}$, $\qquad\qquad y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{4}$, $y'\left(\dfrac{\pi}{4}\right) = 1$.

28.12. $y'' = x + \sin x$, $\qquad\qquad y(0) = -3$, $y'(0) = 0$.

28.13. $y'' = 2\sin x \cos 2x$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

28.14. $y'' = \dfrac{\operatorname{tg} x}{\cos^2 x}$, $\qquad\qquad y(0) = \dfrac{1}{2}$, $y'(0) = 0$.

28.15. $y''' = e^{\frac{x}{2}} + 1$, $\qquad\qquad y(0) = 8$, $y'(0) = 5$, $y''(0) = 2$.

28.16. $y'' = \dfrac{x}{e^{2x}}$, $\qquad\qquad y(0) = \dfrac{1}{4}$, $y'(0) = -\dfrac{1}{4}$.

28.17. $y'' = \sin^2 3x$, $\qquad\qquad y(0) = -\dfrac{\pi^2}{16}$, $y'(0) = 0$.

28.18. $y''' = x\sin x$, $\qquad\qquad y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$.

28.19. $y''' \sin^4 x = \sin 2x$, $\qquad\qquad y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$, $y'\left(\dfrac{\pi}{2}\right) = 1$, $y''\left(\dfrac{\pi}{2}\right) = -1$.

28.20. $y'' = \cos x + e^{-x}$, $\qquad\qquad y(0) = -e^{-\pi}$, $y'(0) = 1$.

28.21. $y'' = \sin^3 x$, $\qquad\qquad y\left(\dfrac{\pi}{2}\right) = -\dfrac{7}{9}$, $y'\left(\dfrac{\pi}{2}\right) = 0$.

28.22. $y''' = \sqrt{x} - \sin 2x$, $\qquad\qquad y(0) = -\dfrac{1}{8}$, $y'(0) = \dfrac{1}{8}\cos 2$, $y''(0) = \dfrac{1}{2}$.

28.23. $y'' = \dfrac{1}{\cos^2 \dfrac{x}{2}}$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

28.24. $y'' = 2\sin x \cos^2 x$, $\qquad\qquad y(0) = -\dfrac{5}{9}$, $y'(0) = -\dfrac{2}{3}$.

28.25. $y'' = \sin^2 x \cos x$, $\qquad\qquad y(0) = \dfrac{1}{9}$, $y'(0) = 1$.

28.26. $y'' = \operatorname{arctg} x$, $\qquad\qquad y(0) = y'(0) = 0$.

28.27. $y'' = -2\cos x \cos 2x$, $\qquad\qquad y(0) = \dfrac{2}{3}$, $y'(0) = \dfrac{2}{3}$.

28.28. $y'' = x - \ln x$, $\qquad\qquad y(1) = -\dfrac{5}{12}$, $y'(1) = \dfrac{3}{2}$.

28.29. $y'' = \dfrac{1}{x^2}$, $\qquad\qquad y(1) = 3$, $y'(1) = 1$.

28.30. $y''' = \cos 4x$, $\qquad\qquad y(0) = 2$, $y'(0) = \dfrac{15}{16}$, $y''(0) = 0$.

Завдання 29. Розв'язати диференціальне рівняння.

29.1. $(1 - x^2) y'' - xy' = 2$. $\qquad$ **29.2.** $2xy'y'' = (y')^2 - 1$.

29.3. $x^3 y'' + x^2 y' = 1$. $\qquad$ **29.4.** $y'' + y' \operatorname{tg} x = \sin 2x$.

29.5. $y'' x \ln x = y'$. $\qquad$ **29.6.** $xy'' - y' = x^2 e^x$.

29.7. $y'' x \ln x = 2y'$. $\qquad$ **29.8.** $x^2 y'' + xy' = 1$.

29.9. $y'' = -\dfrac{x}{y}$. $\qquad$ **29.10.** $xy'' = y'$.

29.11. $y'' = y' + x$. $\qquad$ **29.12.** $xy'' = y' + x^2$.

29.13. $xy'' = y' \ln\left(\dfrac{y'}{x}\right)$. $\qquad$ **29.14.** $xy'' + y' = \ln x$.

29.15. $y'' \operatorname{tg} x = y' + 1$. $\qquad$ **29.16.** $y'' + 2x(y')^2 = 0$.

29.17. $2xy'y'' = (y')^2 + 1$. $\qquad$ **29.18.** $y'' - \dfrac{y'}{x-1} = x(x-1)$.

29.19. $y''' + y'' tg x = \sec x$. $\qquad$ **29.20.** $y'' - 2y' \operatorname{ctg} x = \sin^3 x$.

29.21. $y'' + 4y' = 2x^2$. $\qquad$ **29.22.** $xy'' - y' = 2x^2 e^x$.

29.23. $x(y'' + 1) + y' = 0$. $\qquad$ **29.24.** $y'' + 4y' = \cos 2x$.

29.25. $y'' + y' = \sin x$. $\qquad$ **29.26.** $x^2 y'' = (y')^2$.

29.27. $2xy'' y' = (y')^2 - 4$. $\qquad$ **29.28.** $y''' x \ln x = y''$.

29.29. $y'' \operatorname{ctg} x + y' = 2$. $\qquad$ **29.30.** $(1 + x^2) y'' = 2xy$.

Завдання 30. Знайти розв'язок диференціального рівняння, що задовольняє задані початкові умови.

30.1. $y'' = y' e^y$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

30.2. $(y')^2 + 2yy'' = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.3. $yy'' + (y')^2 = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.4. $y'' + 2y(y')^3 = 0$, $\qquad\qquad y(0) = 2$, $y'(0) = \dfrac{1}{3}$.

30.5. $y'' \operatorname{tg} y = 2(y')^2$, $\qquad\qquad y(1) = \dfrac{\pi}{2}$, $y'(1) = 2$.

30.6. $2yy'' = (y')^2$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.7. $yy'' - (y')^2 = y^4$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.8. $y'' = -\dfrac{1}{2y^3}$, $\qquad\qquad y(0) = \dfrac{1}{2}$, $y'(0) = \sqrt{2}$.

30.9. $y'' = 1 - (y')^2$, $\qquad\qquad y(0) = 0$, $y'(0) = 0$.

30.10. $\left(y''\right)^2 = y'$, $\qquad\qquad y(0) = \dfrac{2}{3}$, $y'(0) = 1$.

30.11. $2yy'' - \left(y'\right)^2 + 1$, $\qquad\qquad y(0) = 2$, $y'(0) = 1$.

30.12. $y'' = 2 - y$, $\qquad\qquad y(0) = 2$, $y'(0) = 2$.

30.13. $y'' = \dfrac{1}{y^3}$, $\qquad\qquad y(0) = 1$, $y'(0) = 0$.

30.14. $yy'' - 2\left(y'\right)^2 = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 2$.

30.15. $y'' = y' + \left(y'\right)^2$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

30.16. $y'' + \dfrac{2}{1-y}\left(y'\right)^2 = 0$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

30.17. $y''(1+y) = 5\left(y'\right)^2$, $\qquad\qquad y(0) = 0$, $y'(0) = 1$.

30.18. $y''(2y+3) - 2\left(y'\right)^2 = 0$, $\qquad\qquad y(0) = 0$, $y'(0) = 3$.

30.19. $4\left(y''\right)^2 = 1 + \left(y'\right)^2$, $\qquad\qquad y(0) = 1$, $y'(0) = 0$.

30.20. $2\left(y'\right)^2 = (y-1)y''$, $\qquad\qquad y(0) = 2$, $y'(0) = 2$.

30.21. $1 + \left(y'\right)^2 = yy''$, $\qquad\qquad y(0) = 1$, $y'(0) = 0$.

30.22. $y'' + y\left(y'\right)^3 = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 2$.

30.23. $yy'' - \left(y'\right)^2 = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 2$.

30.24. $yy'' - \left(y'\right)^2 = y^2 \ln y$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.25. $y'' - \dfrac{(1 + \ln y)\left(y'\right)^2}{y(1 - \ln y)} = 0$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.26. $y''(1+y) = \left(y'\right)^2 + y'$, $\qquad\qquad y(0) = 2$, $y'(0) = 2$.

30.27. $y'' = \dfrac{y'}{\sqrt{y}}$, $\qquad\qquad y(0) = 1$, $y'(0) = 2$.

30.28. $y^3 y' y'' + 1 = 0$, $\qquad\qquad y(1) = 1$, $y'(1) = \sqrt[3]{\dfrac{3}{2}}$.

30.29. $yy'' - 2yy' \ln y = \left(y'\right)^2$, $\qquad\qquad y(0) = 1$, $y'(0) = 1$.

30.30. $y'' = \dfrac{1}{\sqrt{y}}$, $\qquad\qquad y(0) = y'(0) = 0$.