

Диференціальні рівняння

Завдання 25. Розв'язати диференціальне рівняння.

25.1. $xy' = 1 + y^2$.

25.2. $yy'\sqrt{1+x^2} = x\sqrt{1+y^2}$.

25.3. $y' = \frac{x^2y + y}{\sqrt{4+y^2}}$.

25.4. $x + xy + y'(y + xy) = 0$.

25.5. $(y - x^2y)' = 4x - 5xy^2$.

25.6. $y' = \frac{y}{\sqrt{x^2 + 1}}$.

25.7. $y'tgx = y$.

25.8. $(e^{2x} + 5)y' = ye^{2x}$.

25.9. $e^{2x}(2y - 1)y' = y$.

25.10. $(x + 4)y' = y^2 - 1$.

25.11. $(1 + e^x)yy' = e^x$.

25.12. $\sqrt{4 - x^2}y' + xy^2 + x = 0$.

25.13. $(e^x + 8)y' = ye^x$.

25.14. $2x + 2xy^2 + \sqrt{2 - x^2}y' = 0$.

25.15. $y'ctgx = y^4$.

25.16. $y'y\sqrt{1 - x^2} = \sqrt{5 + y^2}$.

25.17. $(2x - xy^2)dx = (y + yx^2)dy$.

25.18. $y \ln y + xy' = 0$.

25.19. $xy' + y = y^2$.

25.20. $\sqrt{1 - x^2}y' + xy^2 + x = 0$.

25.21. $xy' - 2y = yx^3$.

25.22. $xy' = y(1 + \ln y)$.

25.23. $(3 + e^x)yy' = e^x$.

25.24. $\sqrt{3 + y^2} + \sqrt{1 - x^2}yy' = 0$.

25.25. $y' \sin x = y \ln y$.

25.26. $(1 + e^x)yy' = e^x$.

25.27. $yy' = e^x(4 + y^2)$.

25.28. $\sqrt{4 - x^2}y' = 3x + xy^2$.

25.29. $y'ctgy = x^3$.

25.30. $y' = 2\sqrt{y} \ln x$.

Завдання 26. Знайти загальний розв'язок диференціального рівняння.

26.1. $y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2$.

26.2. $y' = \frac{y}{x} + \sin \frac{y}{x}$.

26.3. $y' = \frac{x + y}{x - y}$.

26.4. $y' = \frac{y^2}{x^2} - \frac{y}{x}$.

26.5. $y' = \frac{y^2}{x^2} + 5\frac{y}{x} + 8$.

26.6. $y' = \frac{y}{x} + \operatorname{tg} \frac{y}{x}$.

26.7. $xy' \cos \frac{y}{x} = y \cos \frac{y}{x} - x$.

26.8. $y' = \frac{x + 2y}{2x - y}$.

26.9. $y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4$.

26.10. $xy' = y \left(1 + \ln \frac{y}{x} \right)$.

26.11. $xy' + 2\sqrt{xy} = y$.

26.12. $xy' = \sqrt{2x^2 + y^2} + y$.

26.13. $y' = \frac{y^2}{x^2} + 3\frac{y}{x} + 5$.

26.14. $xy' - y = x \operatorname{tg} \frac{y}{x}$.

$$26.15. xy' = xe^{-\frac{y}{x}} + y.$$

$$26.16. xy' = y + 2x \sin^2 \frac{3y}{x}.$$

$$26.17. y' = \frac{y^2}{x^2} + 7\frac{y}{x} + 9.$$

$$26.18. y' = \frac{x^2 + xy + y^2}{x^2}.$$

$$26.19. xy' = \sqrt{x^2 - y^2} + y.$$

$$26.20. xy' - y = y \ln \frac{y}{x}.$$

$$26.21. y' = 2\frac{y^2}{x^2} + 5\frac{y}{x} + 1.$$

$$26.22. xy' = y + 2x \operatorname{tg} \frac{3y}{x}.$$

$$26.23. xy' = 3\sqrt{x^2 - y^2} + y.$$

$$26.24. xy' = y + x \cos^2 \frac{y}{x}.$$

$$26.25. xy' = y + x \sin^2 \frac{2y}{x}.$$

$$26.26. y' = \frac{y^2}{x^2} + 9\frac{y}{x} + 16.$$

$$26.27. y' = \frac{y^2}{x^2} + 3\frac{y}{x} + 1.$$

$$26.28. xy' = 4\sqrt{x^2 + y^2} + y.$$

$$26.29. y' = 3 \cos^2 \frac{2y}{x} + \frac{y}{x}.$$

$$26.30. y' = \frac{y^2}{x^2} + 5\frac{y}{x} + 4.$$

Завдання 27. Знайти розв'язок диференціального рівняння, що задовольняє задану початкову умову.

$$27.1. y' - \frac{y}{x} = x^2,$$

$$y(1) = 0.$$

$$27.2. y' - y \operatorname{ctg} x = 2x \sin x,$$

$$y\left(\frac{\pi}{2}\right) = 0.$$

$$27.3. y' + y \cos x = \frac{1}{2} \sin 2x,$$

$$y(0) = 0.$$

$$27.4. y' + y \operatorname{tg} x = \cos^2 x,$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{2}.$$

$$27.5. y' - \frac{y}{x+2} = x^2 + 2x,$$

$$y(-1) = \frac{3}{2}.$$

$$27.6. y' - \frac{x}{x+1} y = e^x (x+1),$$

$$y(0) = 1.$$

$$27.7. y' - \frac{y}{x} = x \sin x,$$

$$y\left(\frac{\pi}{2}\right) = 1.$$

$$27.8. y' - \frac{2x-5}{x^2} y = 5,$$

$$y(2) = 4.$$

$$27.9. y' + \frac{y}{2x} = x^2,$$

$$y(1) = 1.$$

$$27.10. y' + \frac{2x}{1+x^2} y = \frac{2x^2}{1+x^2},$$

$$y(0) = \frac{2}{3}.$$

$$27.11. y' + \frac{y}{x} = \sin x,$$

$$y(\pi) = \frac{1}{\pi}.$$

$$27.12. y' + \frac{y}{x} = \frac{x+1}{x} e^x,$$

$$y(1) = e.$$

$$27.13. y' - \frac{y}{x} = -2\frac{\ln x}{x},$$

$$y(1) = 1.$$

- 27.14. $y' + \frac{1-2x}{x^2}y = 1$, $y(1) = 1$.
- 27.15. $y' + \frac{2}{x}y = x^3$, $y(1) = -5/6$.
- 27.16. $y' - \frac{2xy}{1+x^2} = 1+x^2$, $y(1) = 3$.
- 27.17. $y' + \frac{y}{x} = 3x$, $y(1) = 1$.
- 27.18. $y' - \frac{y}{x} = -\frac{12}{x^3}$, $y(1) = 4$.
- 27.19. $y' + \frac{3y}{x} = \frac{2}{x^3}$, $y(1) = 1$.
- 27.20. $y' + 2xy = -2x^3$, $y(1) = e^{-1}$.
- 27.21. $y' + \frac{xy}{2(1-x^2)} = \frac{x}{2}$, $y(0) = \frac{2}{3}$.
- 27.22. $y' + xy = -x^3$, $y(0) = 3$.
- 27.23. $y' - \frac{2}{x+1}y = e^x(x+1)^2$, $y(0) = 1$.
- 27.24. $y' + 2xy = e^{-x^2} \sin x$, $y(0) = 1$.
- 27.25. $y' - \frac{2y}{x+1} = (x+1)^3$, $y(0) = \frac{1}{2}$.
- 27.26. $y' - y \cos x = -\sin 2x$, $y(0) = 3$.
- 27.27. $y' - 4xy = -4x^3$, $y(0) = -\frac{1}{2}$.
- 27.28. $y' - \frac{y}{x} = -\frac{\ln x}{x}$, $y(1) = 1$.
- 27.29. $y' - 3x^2y = \frac{x^2(1+x^3)}{3}$, $y(0) = 0$.
- 27.30. $y' - \frac{y}{x} = -\frac{2}{x^2}$, $y(1) = 1$.