Biosignals and Systems

Basic of Wavelet Transformatoin

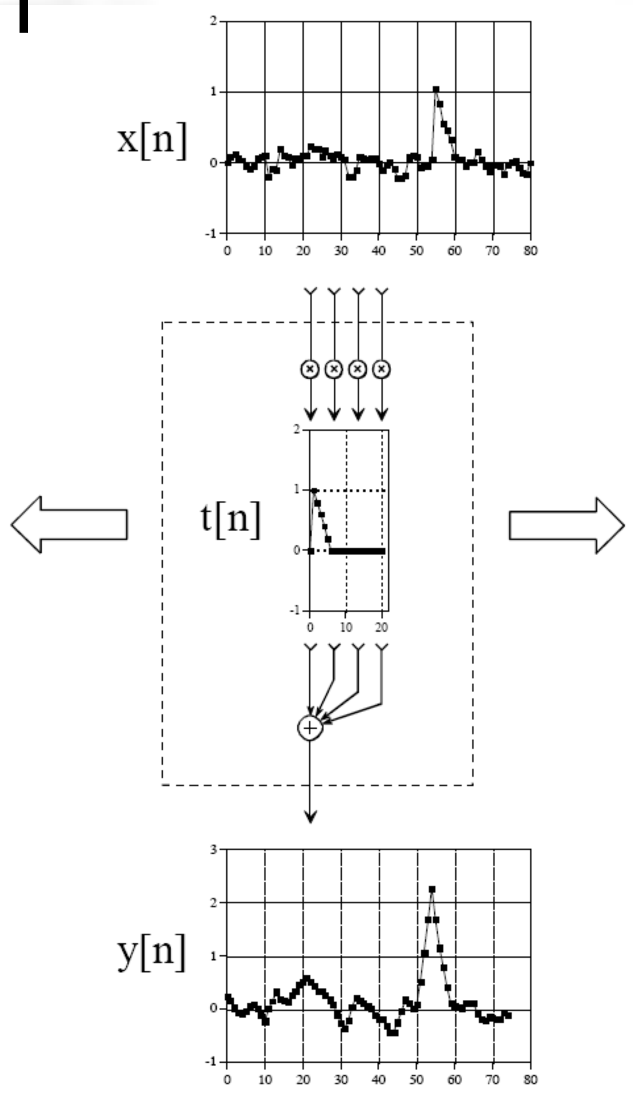
Other Signal Processing Techniques
Correlation

- mathematical operation that is very similar to convolution

- uses two signals to produce a third signal. This third signal is called the **cross-correlation** of the two input signals (i.e.finds similar signals in a signal)

- if a signal is correlated with *itself*, the resulting signal is instead called the **auto-correlation** (i.e.finds periodic parts of a signal)

- Correlation is the *optimal* technique for detecting a known waveform in random noise.



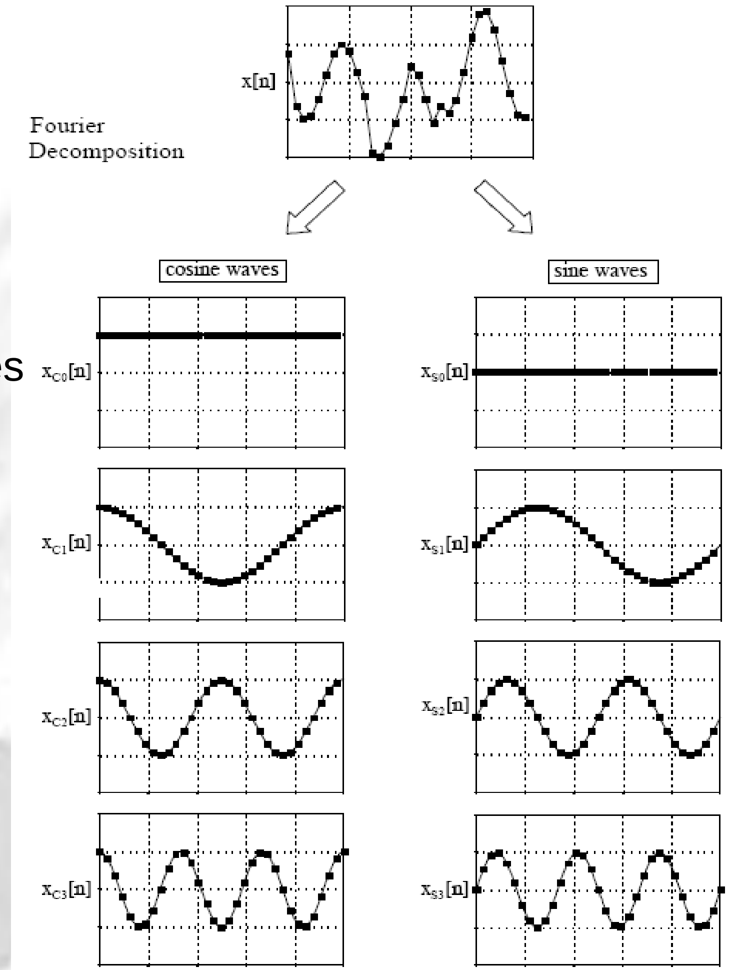
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, '' **Discrete-Time Signal Processing**'', Prentice-Hall, Inc.1999,1989
[11] http://www.dspguide.com

Other Signal Processing Techniques Discrete Fourier Transform (DFT)

- Decomposition into sine- and cosine waves

$$c_k = \cos(2\pi ki / N)$$
$$s_k = \sin(2\pi ki / N)$$

k .. base function
i .. sample index
N .. number of samples



- Finds frequency components of (periodic) signals
- Frequencies up to F/2

[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, ‘‘ **Discrete-Time Signal Processing**’’, Prentice-Hall, Inc.1999,1989
[11] http://www.dspguide.com

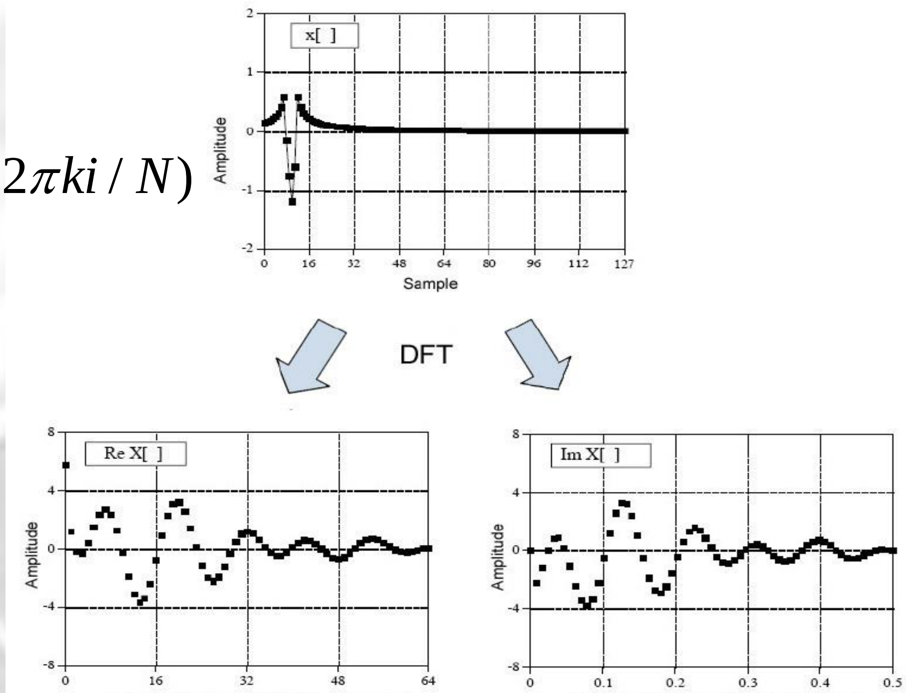
Other Signal Processing Techniques
Discrete Fourier Transform(DFT)

- Inverse Transform:

$$\mathrm{Re}\,X[k]=\sum_{i=0}^{N-1}x[i]\cos(2\pi ki/N)$$

$$lmX[k]=\sum_{i=0}^{N-1}x[i]\sin(2\pi ki/N)$$

$$x[i]=\sum_{k=0}^{N/2}\mathrm{Re}\,X[k]\cos(2\pi ki/N)+\sum_{k=0}^{N/2}lmX[k]\sin(2\pi ki/N)$$



- FFT-Algorithms

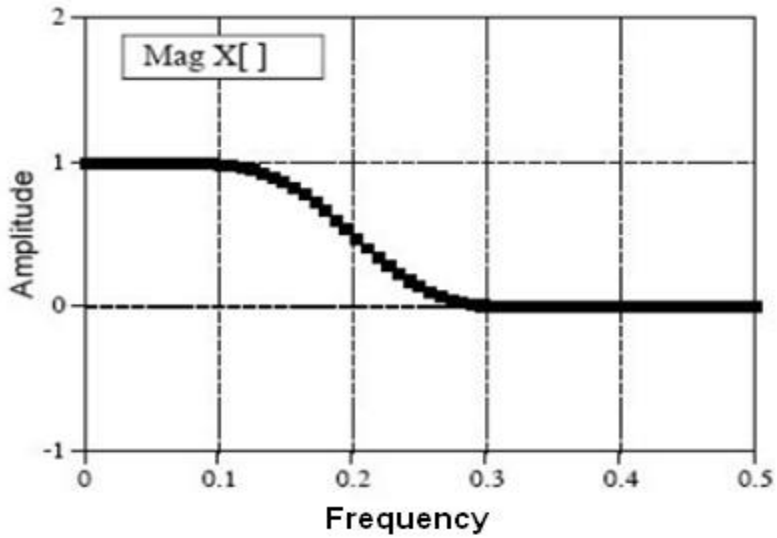
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, ‘‘ **Discrete-Time Signal Processing**’’, Prentice-Hall, Inc.1999,1989
[11] http://www.dspguide.com

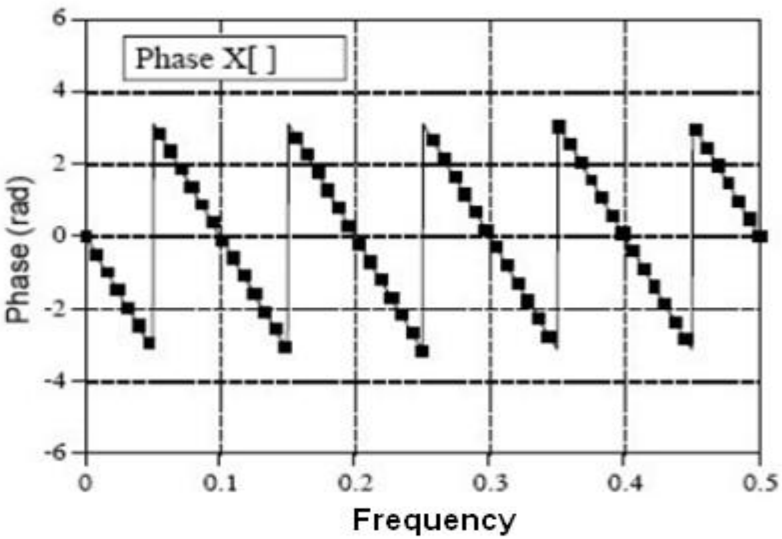
Other Signal Processing Techniques Discrete Fourier Transform(DFT)

- Calculation of Magnitude and Phase response:

$$MagX[k] = \sqrt{(\text{Re}\, X[k]^2 + \text{Im}\, X[k]^2)}$$

$$PhaseX[k] = \arctan\left(\frac{\text{Im}\, X[k]}{\text{Re}\, X[k]}\right)$$

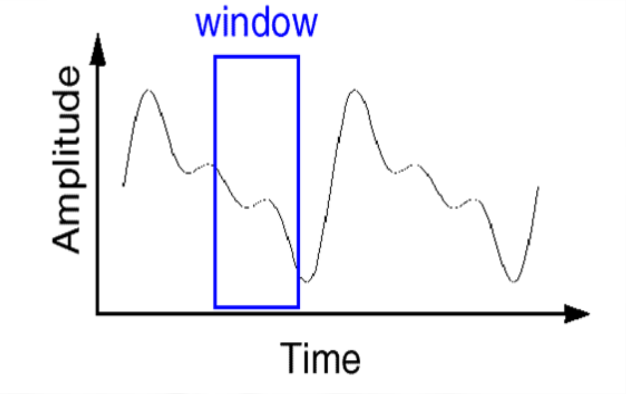


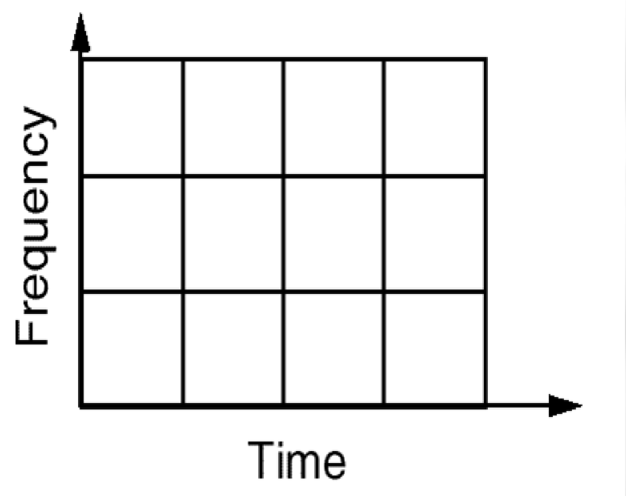


[2] Steven T.Karris, '' **Signals and Systems with MATLAB Computing and Simulink Modeling**'', Fourth Edition 2008
[11] http://www.dspguide.com

Short Time Fourier Analysis

- In order to analyze small section of a signal, Denis Gabor (1946), developed a technique, based on the FT and using *windowing* : STFT
- A compromise between time-based and frequency-based views of a signal.
- both time and frequency are represented in limited precision.
- The precision is determined by the size of the window.
- Once you choose a particular size for the time window - it will be the same for all frequencies.
- Many signals require a more flexible approach - so we can vary the window size to determine more accurately either time or frequency.

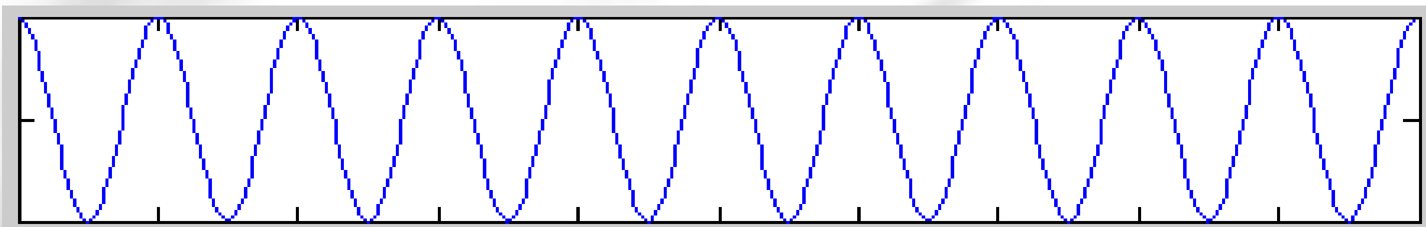




[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, ‘‘ **Discrete-Time Signal Processing**’’, Prentice-Hall, Inc.1999,1989
[11] http://www.dspguide.com
[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

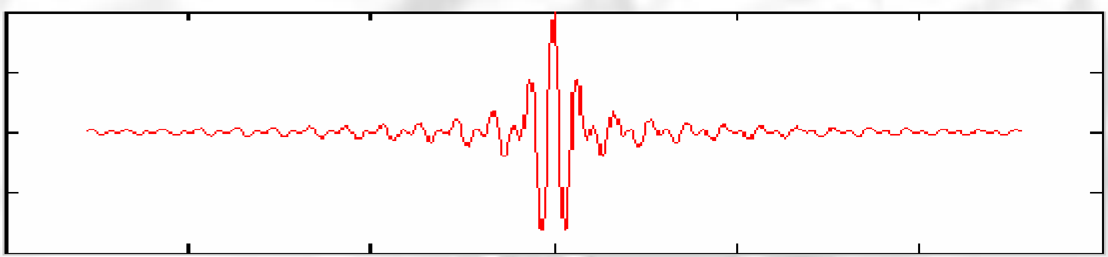
Fourier Analysis-Wavelet Analysis

- Fourier Analysis is based on an indefinitely long cosine wave of a specific Frequency



time, t

- Wavelet Analysis is based on an short duration wavelet of a specific center frequency

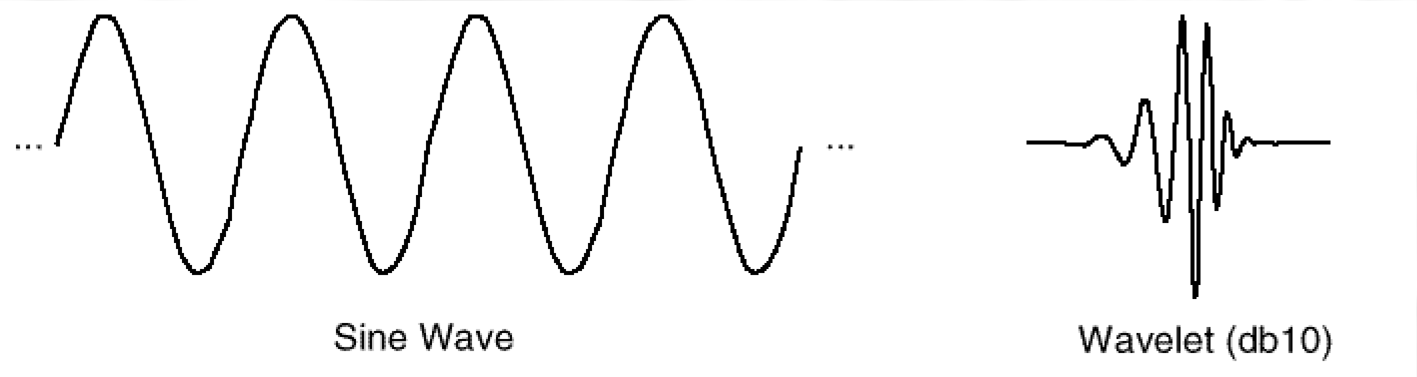


time, t

[2] Steven T.Karris, ‘‘ **Signals and Systems with MATLAB Computing and Simulink Modeling**’’, Fourth Edition, 2008
[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

What is Wavelet Analysis ?

- A wavelet is a waveform of effectively limited duration that has an average value of zero.



[2] Steven T.Karris, ‘‘ **Signals and Systems with MATLAB Computing and Simulink Modeling**’’, Fourth Edition, 2008

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet's properties

- Short time localized waves with zero integral value.
- Possibility of time shifting.
- Flexibility.

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

The Continuous Wavelet Transform (CWT)

A mathematical representation of the Fourier transform:

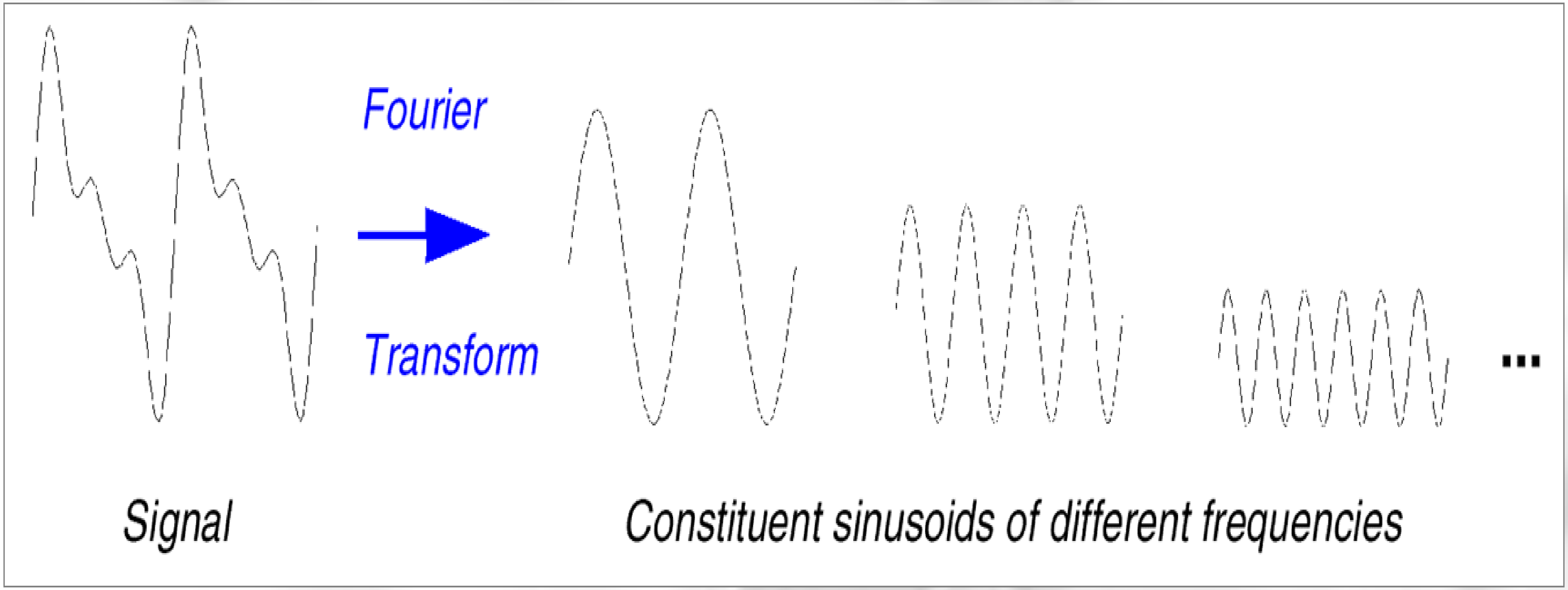
$$F_w = \int_{-\infty}^{+\infty} f(t) e^{-iwt} \cdot dt$$

the sum over all time of the signal *f(t)* multiplied by a complex exponential, and the result is the Fourier coefficients F(w) .

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation

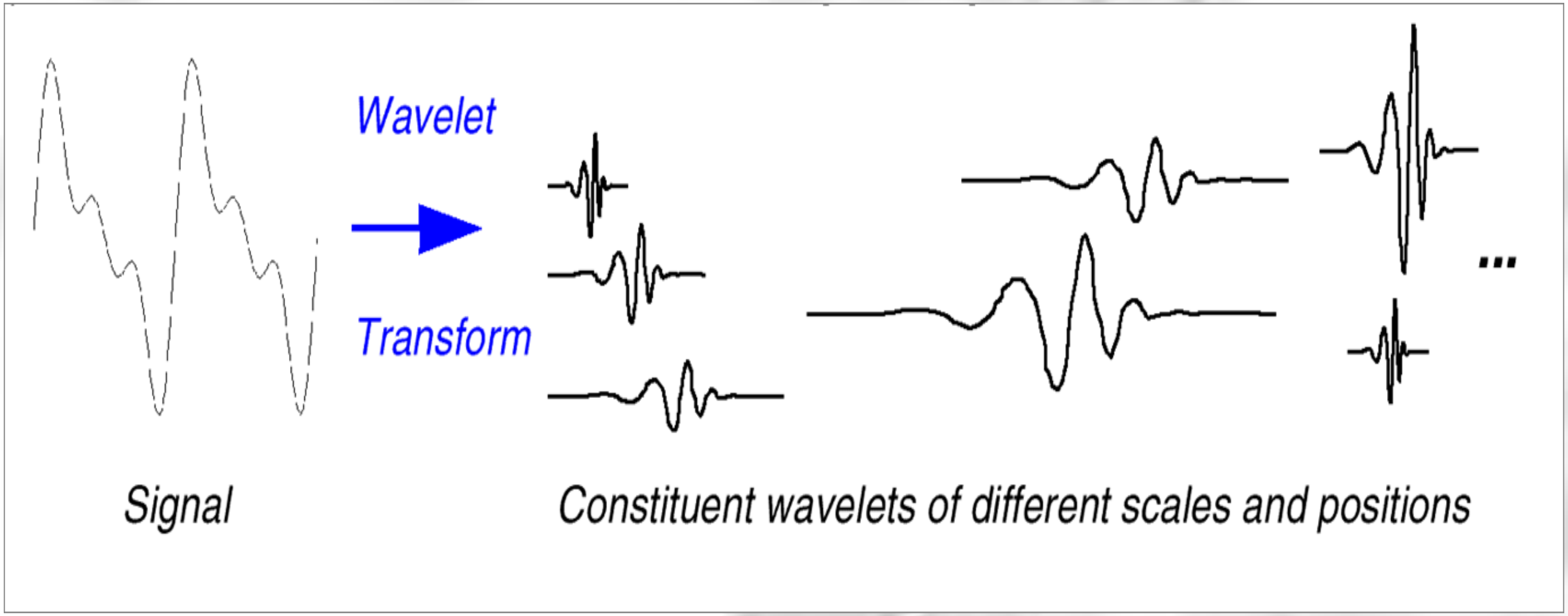
Those coefficients, when multiplied by a sinusoid of appropriate frequency, yield the constituent sinusoidal component of the original signal:



[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation

- And the result of the CWT are Wavelet coefficients
- Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelet of the original signal:



[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

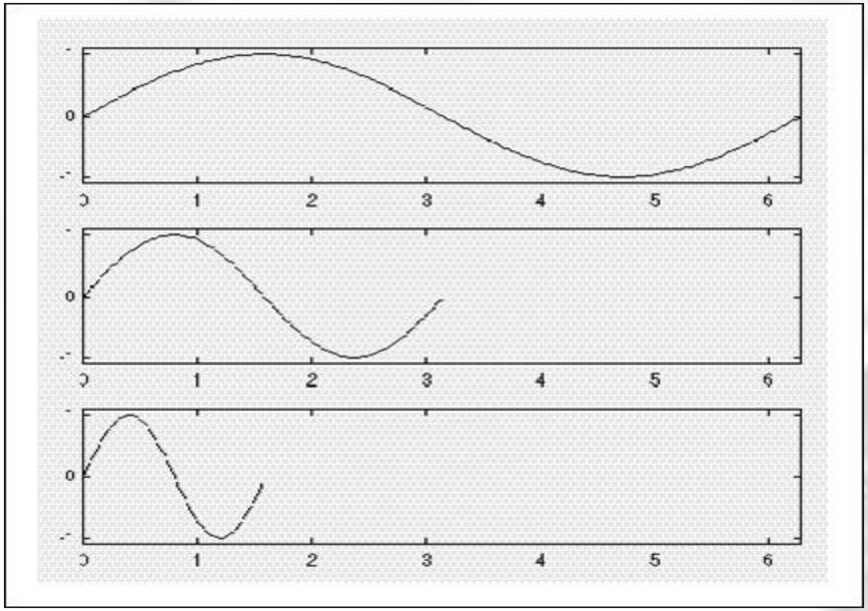
Wavelet Transformation-Equations

- Wavelet Transform $$\gamma(s,\tau) = \int f(t)\psi^{*}_{S,T}(t)dt$$
- Inverse Wavelet Transform $$f(t) = \iint \gamma(s,\tau)\psi_{S,T}(t)d\tau ds$$
- All wavelet derived from *mother wavelet* $$\psi_{S,T}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-\tau}{s})$$

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation-Scaling

- Wavelet analysis produces a time-scale view of the signal.
- *Scaling* means stretching or compressing of the signal.
- scale factor (a) for sine waves:



$$f(t) = \sin(t); a = 1$$

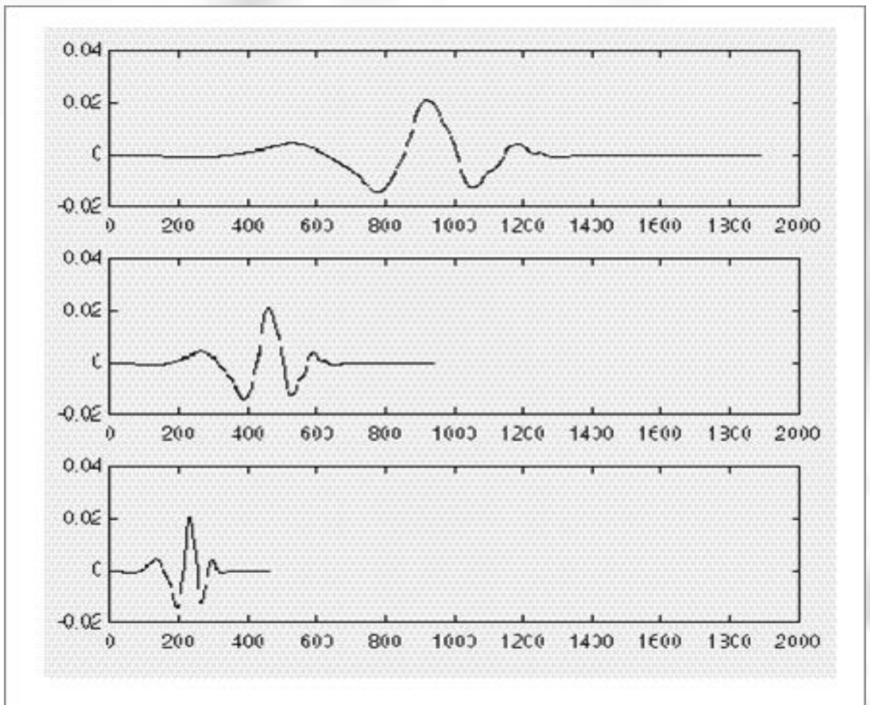
$$f(t) = \sin(2t); a = \frac{1}{2}$$

$$f(t) = \sin(4t); a = \frac{1}{4}$$

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation-Scaling

- Scale factor works exactly the same with wavelets:



$$f(t) = \Psi(t); a = 1$$

$$f(t) = \Psi(2t); a = \frac{1}{2}$$

$$f(t) = \Psi(4t); a = \frac{1}{4}$$

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[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation- Wavelet function

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}} \Psi\left(\frac{x-b}{a}\right)$$

- b- shift coefficient
- a- scale coefficient

$$\Psi_{a,b_x,b_y}(x,y) = \frac{1}{|a|} \Psi\left(\frac{x-b_x}{a}, \frac{y-b_y}{a}\right)$$

- 2D function

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation-Wavelet function

normalization

shift in time

$$\psi_{s,T}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

wavelet with
scale, s and time, τ

Mother wavelet

change in scale:
big s means long
wavelength

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation

I'm going to ignore the complex conjugate from now on, assuming that we're using real wavelets

time-series

$$\gamma(s,\tau) = \int f(t)\psi^{*}_{s,T}(t)\,dt$$

coefficient of wavelet with scale, s and time, τ

complex conjugate of wavelet with scale, s and time, τ

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation-Wavelets examples
Dyadic transform

- For easier calculation we can to discrete continuous signal.
- We have a grid of discrete values that called dyadic grid .
- Important that wavelet functions compact (e.g. no over-calculatings)

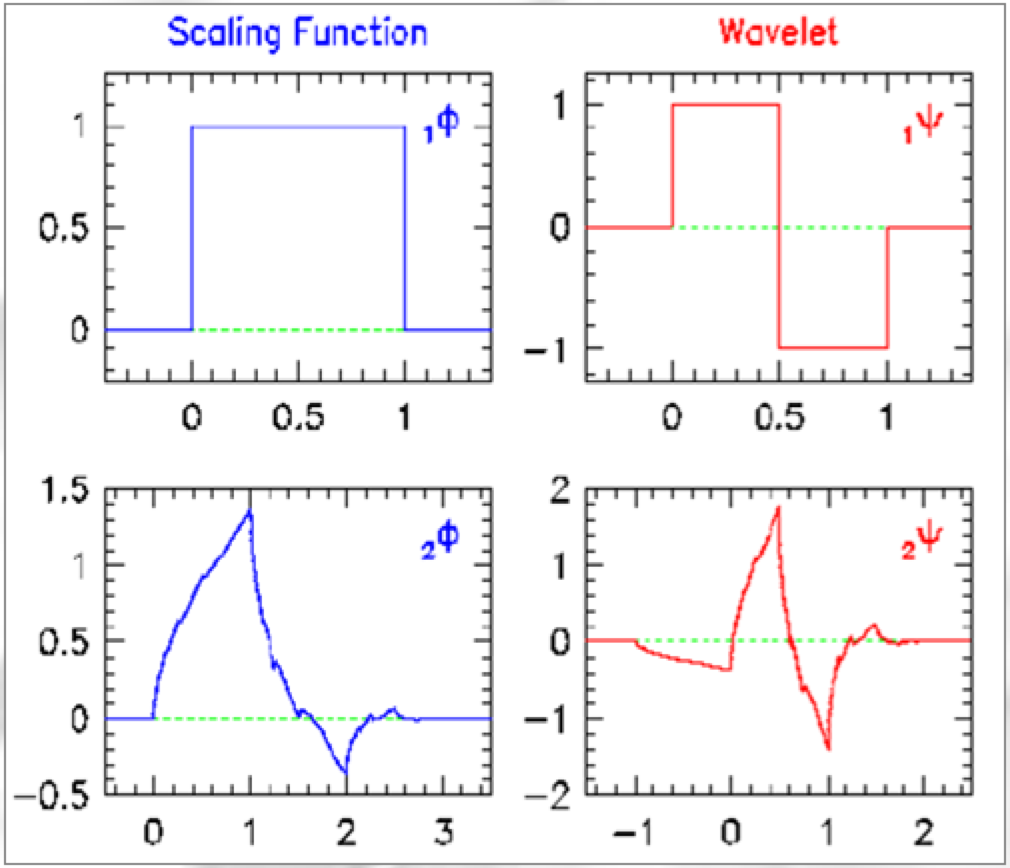
$$a = 2^j$$

$$b = k2^j$$

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation- Wavelet functions examples

- Haar function
- Daubechies function



[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Inverse Wavelet Transform

$$f(t) = \iint \gamma(s,\tau)\psi_{S,T}(t)\,d\tau\,ds$$

time-series

coefficients of wavelets

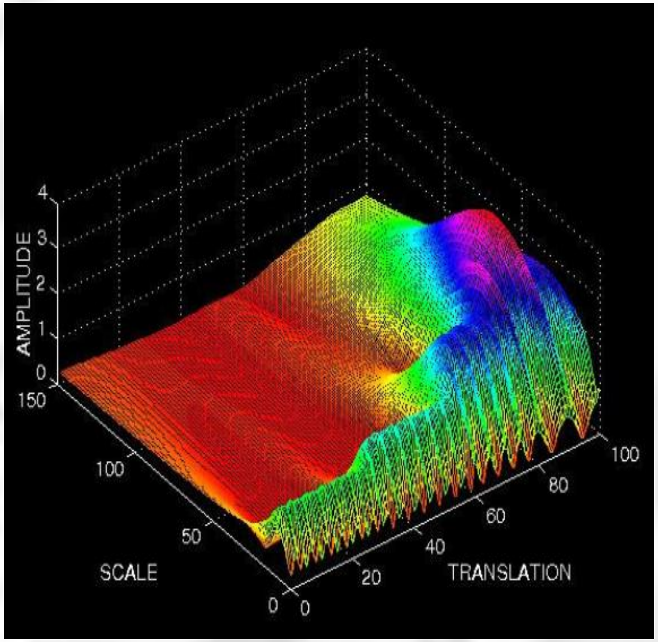
wavelet with scale, s and time, τ

build up a time-series as sum of wavelets of different scales, s, and positions, t

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

Wavelet Transformation

- good frequency resolution at low frequencies and
- good time resolution at high frequencies
- no work-around for the principle of entropy



- scale (s) and translation (t) of the base wavelet
- convolution with the signal
- special wavelets for special purposes

[11] http://www.dspguide.com

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.