The first half of Chapter 3

Time - Domain Analysis
Of
Discrete - Time Systems

3.1. Introduction

3.1-0 Discrete signals and systems

Discrete-Time Signal

A Sequence of numbers

- inherently Discrete-Time
- sampling Continuous-Time signals, ex) $x[n], y[n]$

Discrete-Time System

- Inputs and outputs are Discrete-Time signals.
- A Discrete-Time(DT) signal is a sequence of numbers
- DT system processes a sequence of numbers $x[n]$ to yield another sequence $y[n]$.

3.1. Introduction

Discrete-Time Signal by sampling CT signal

$$x(t) = e^{-t}$$

$$x(nT) = e^{-nT} = e^{-0.1n} = x[n]$$

T = 0.1 : Sampling interval
n : Discrete variable taking on integer values.

- ${}^{C}\!/_{D}, {}^{D}\!/_{C}$ conversion to process CT signal by DT system.

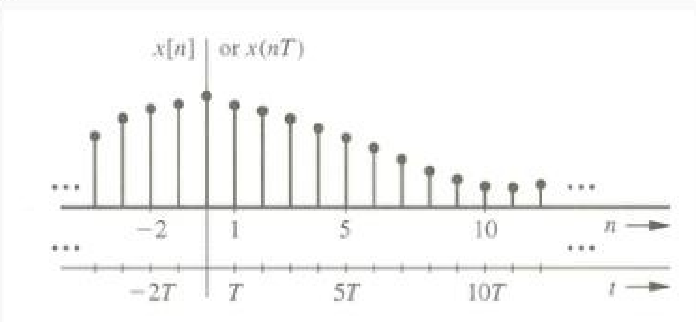


Figure 3.1 A discrete-time signal.

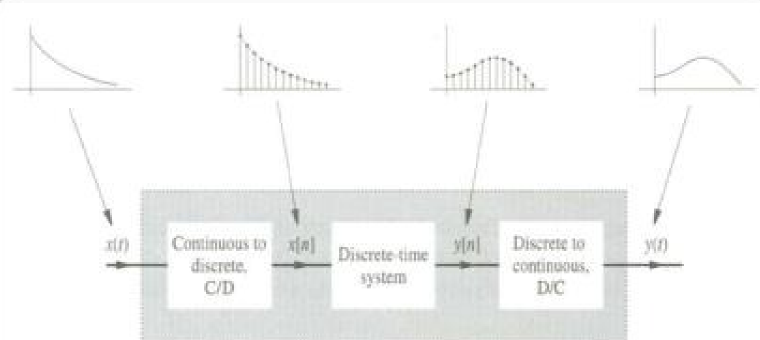


Figure 3.2 Processing a continuous-time signal by means of a discrete-time system.

3.1. Introduction

3.1-1 Size of DT Signal

- Energy Signal

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad (3.1)$$

Signal amp. $\rightarrow 0$ as $|n| \rightarrow \infty \Rightarrow E_x < \infty$

- Power Signal

$$P_x = \lim_{N->\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2 \qquad (3.2)$$

for periodic signals, one period time averaging.

- A DT signal can either be an energy signal or power signal
- Some signals are neither energy nor power signal.

3.1. Introduction

➢Example 3.1 (1)

Find the energy of the signal $x[n] = n$, shown in Fig. 3.3a and the power for the periodic signal $y[n]$ in Fig. 3.3b.

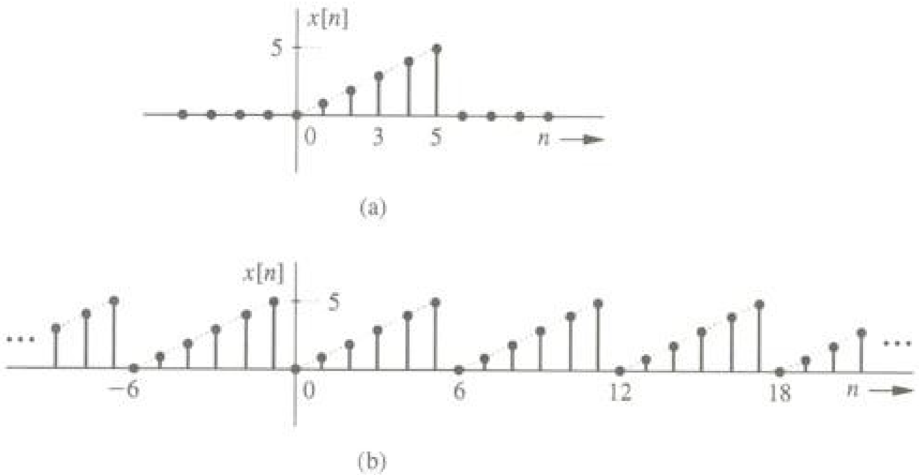


Figure 3.3 (**a**) Energy and (**b**) power computation for a signal.

By definition

$$E_x = \sum_{n=0}^{5} n^2 = 55$$

A periodic signal $x[n]$ with period N_0 is characterized by the fact that

$$x[n] = x[n + N_0]$$

3.1. Introduction

➢Example 3.1 (2)

The smallest value of N_0 for which the preceding equation holds is the *fundamental period*. Such a signal is called N_0 *periodic*. Figure 3.3b shows an example of a periodic signal $y[n]$ of period $N_0 = 6$ because each period contains 6 samples. Note that if the first sample is taken at $n = 0$, the last sample is at $n = N_0 - 1 = 5$, not at $n = N_0 = 6$. Because the signal $y[n]$ is periodic, its power P_y can be found by averaging its energy over one period. Averaging the energy over one period, we obtain

$$P_y = \frac{1}{6}\sum_{n=0}^{5} n^2 = \frac{55}{6}$$

3.2. Useful Signal Operations

- Shifting
  - $x_S[n] = x[n-5]$

Left shift ⇕ Advance	Right shift ⇕ Delay	
$n \to n+M$	$n \to n-M$	$\because M \geq 0$

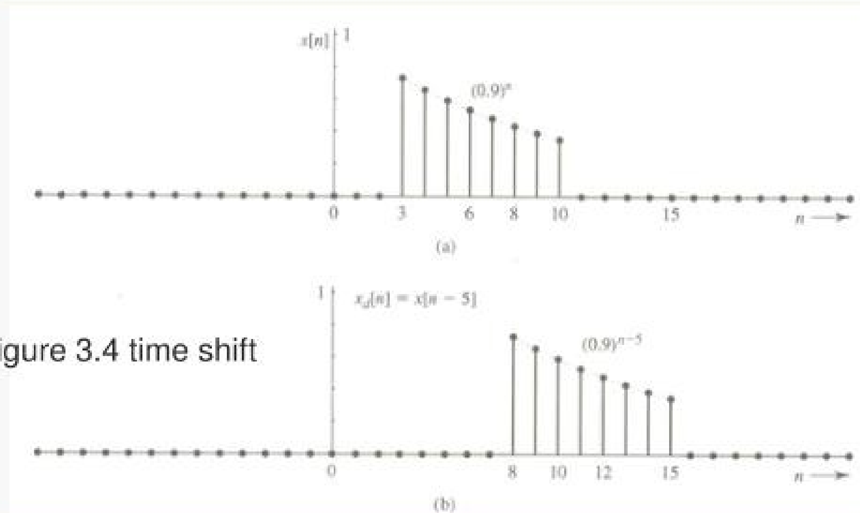


Figure 3.4 time shift

3.2. Useful Signal Operations

➢Time Reversal

- $x_r[n] = x[-n] \qquad , n \to -n$
- anchor point : n = 0

cf) $-x[n]$

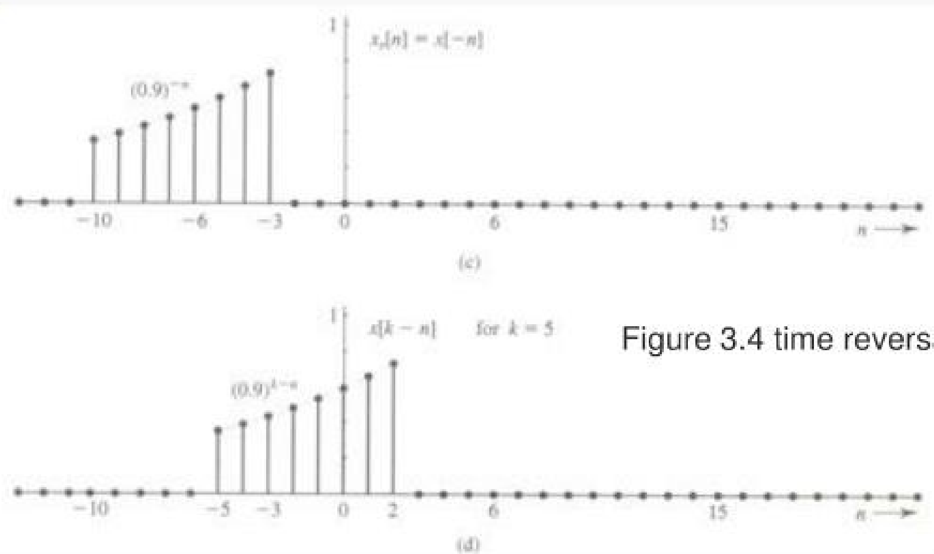


Figure 3.4 time reversal

3.2. Useful Signal Operations

➢Example 3.2

In the convolution operation, discussed later, we need to find the function $x[k-n]$ from $x[n]$.

This can be done in two steps: (i) time-reverse the signal $x[n]$ to obtain $x[-n]$; (ii) now, right-shift $x[-n]$ by k. Recall that right-shifting is accomplished by replacing n with $n-k$. Hence, right-shifting $x[-n]$ by k units is $x[-(n-k)] = x[k-n]$. Figure 3.4d shows $x[5-n]$, obtained this way. We first time-reverse $x[n]$ to obtain $x[-n]$ in Fig. 3.4c. Next, we shift $x[-n]$ by $k=5$ to obtain $x[k-n] = x[5-n]$, as shown in Fig. 3.4d.

In this particular example, the order of the two operations employed is interchangeable. We can first left-shift $x[k]$ to obtain $x[n+5]$. Next, we time-reverse $x[n+5]$ to obtain $x[-n+5] = x[5-n]$. The reader is encouraged to verify that this procedure yields the same result, as in Fig. 3.4d.

$$x[k-n] = x[-n+k] = x[-(n-k)]$$

1. $$x[n] \underset{n=n+k}{\overset{advance(k)}{\rightarrow}} x[n+k] \underset{n=-n}{\overset{reverse}{\rightarrow}} x[-n+k]$$

2. $$x[n] \underset{n=-n}{\overset{reverse}{\rightarrow}} x[-n] \underset{n=n-k}{\overset{delay(k)}{\rightarrow}} x[-(n-k)]$$

3.2. Useful Signal Operations

- Sampling Rate Alteration : Decimation and Interpolation
  - decimation (down sampling)

$$x_d[n] = x[Mn] \quad \text{, M must be integer values} \qquad (3.3)$$

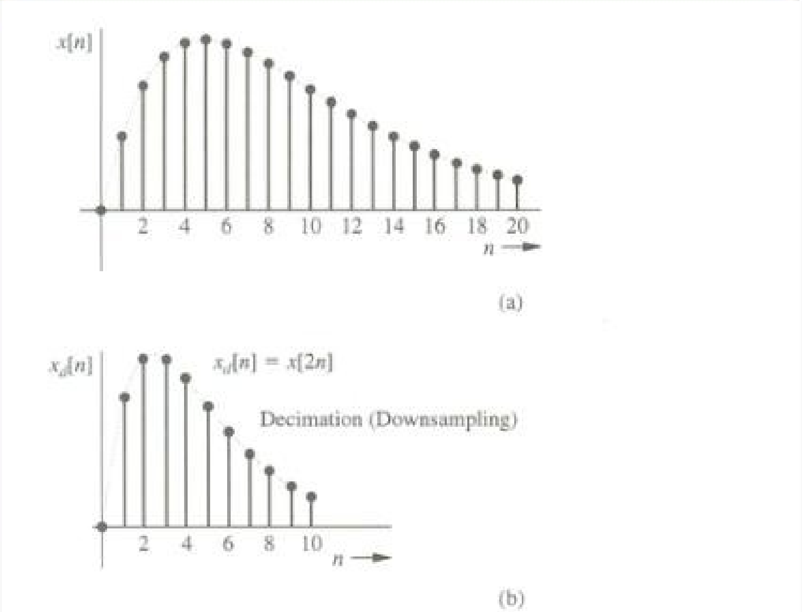


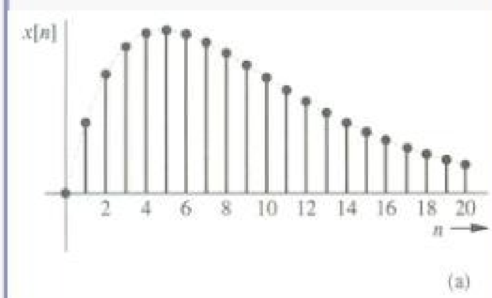
Figure 3.5 Compression (decimation)

3.2. Useful Signal Operations

- interpolation (expanding)

$$x_e[n] = \begin{cases} x[n/L] & n = 0, \quad \pm L, \quad \pm 2L, \ldots, \\ 0 & otherwise \end{cases}$$

L must be integer value more than 2. (3.4)



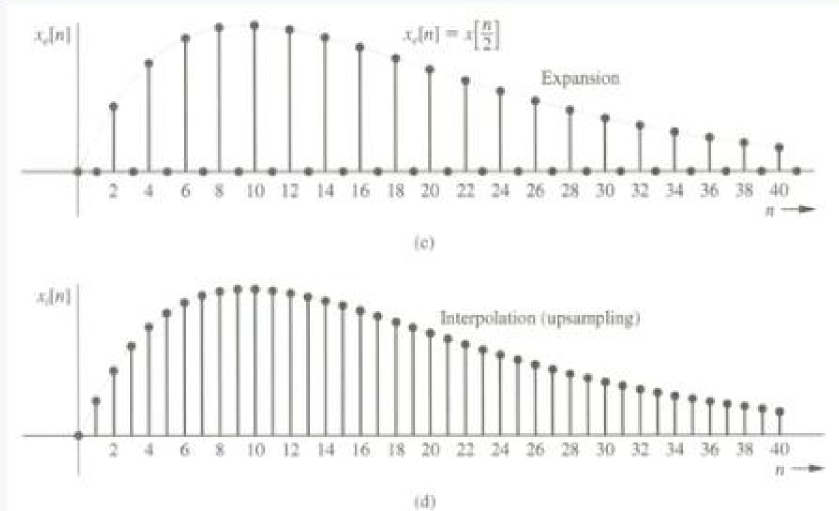


Figure 3.5 expansion (interpolation) of a signal.

3.3. Some Useful DT Signal Models

3.3-1 DT Impulse Function

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \qquad (3.5)$$

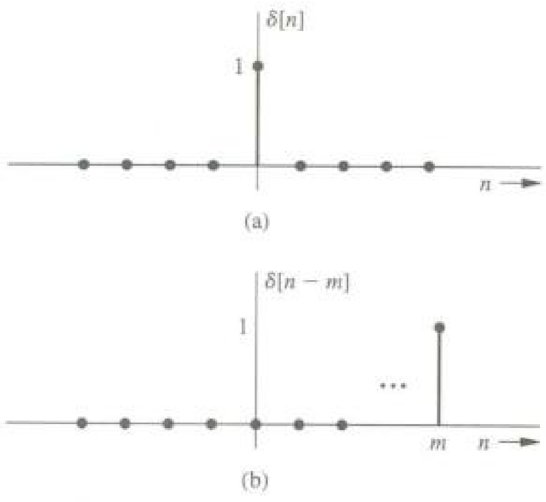


Figure 3.6 Discrete-time impulse function: (**a**) unit impulse sequence and (**b**) shifted impulse sequence.

3.3. Some Useful DT Signal Models

3.3-2 DT Unit Stop Function

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad (3.6)$$

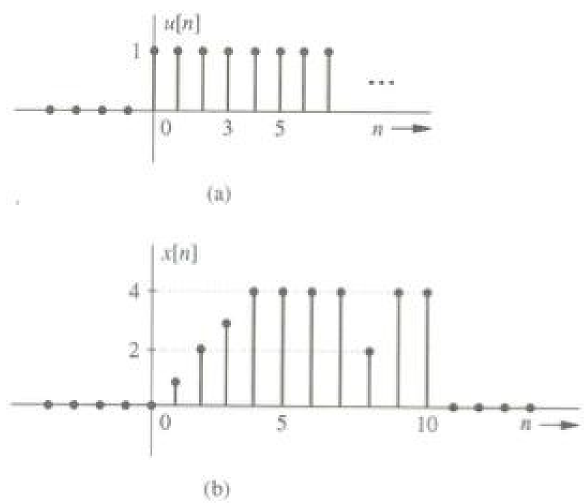


Figure 3.7 **(a)** A discrete-time unit step function $u[n]$ and **(b)** its application.

3.3. Some Useful DT Signal Models

➢Example 3.3 (1)

Describe the signal $x[n]$ shown in Fig. 3.7b by a single expression valid for all n.

There are many different ways of viewing $x[n]$. Although each way of viewing yields a different expression, they are all equivalent. We shall consider here just one possible expression.

The signal $x[n]$ can be broken into three components: (1) a ramp component $x_1[n]$ from $n = 0$ to 4, (2) a step component $x_2[n]$ from $n = 5$ to 10, and (3) an impulse component $x_3[n]$ represented by the negative spike at $n = 8$. Let us consider each one separately.

We express $x_1[n] = n(u[n] - u[n-5])$ to account for the signal from $n = 0$ to 4. Assuming that the spike at $n = 8$ does not exist, we can express $x_2[n] = u[n-5] - u[n-11]$ to account for the signal from $n = 5$ to 10. Once these two components have been added, the only part that is unaccounted for is a spike of amplitude -2 at $n = 8$, which can be represented by $x_3[n] = -2\delta[n-8]$. Hence

$$\begin{aligned} x[n] &= x_1[n] + x_2[n] + x_3[n] \\ &= n(u[n] - u[n-5]) + 4(u[n-5] - u[n-11]) - 2\delta[n-8] \end{aligned}$$

for all n

3.3. Some Useful DT Signal Models

➢Example 3.3 (2)

We stress again that the expression is valid for all values of n. The reader can find several other equivalent expressions for $x[n]$. For example, one may consider the step function from $n = 0$ to 10, subtract a ramp over the range $n = 0$–3, and subtract the spike. You can also play with breaking n into different ranges for your expression.

3.3. Some Useful DT Signal Models

3.3-3 DT Exponential γ^n

- CT Exponential $e^{\lambda t}$

$$e^{\lambda t} = \gamma^t \qquad (\; r = e^{\lambda} \text{ or } \lambda = \ln(\gamma) \;)$$

ex) $e^{-0.3t} = (0.7408)^t \qquad 4^t = e^{1.386t}$

- $e^{\lambda n} = \gamma^n \qquad (\; \gamma = e^{\lambda} \text{ or } \lambda = \ln(\gamma) \;)$

- Nature of γ^n

$$e^{\lambda n} = e^{(a+jb)n} = (e^a e^{jb})^n = \gamma^n$$

$$|\gamma| = |e^a||e^{jb}| = e^a 1 = e^a$$

3.3. Some Useful DT Signal Models

	λ plane	γ plane
$a<0$ exp. dec.	LHP	Inside the unit circle
$a>0$ exp. inc.	RHP	Outside the unit circle
$a=0$ osc.	Imaginary axis	Unit circle

- $\gamma^{-n} = \left(\frac{1}{\gamma}\right)^n$

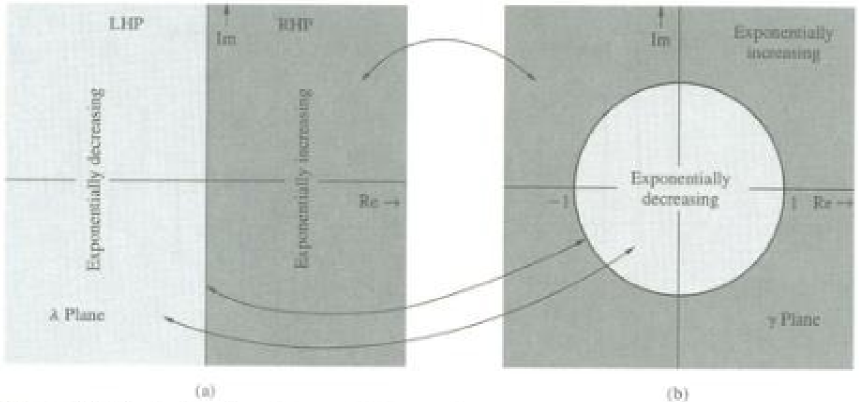


Figure 3.8 The λ plane, the γ plane, and their mapping.

3.3. Some Useful DT Signal Models

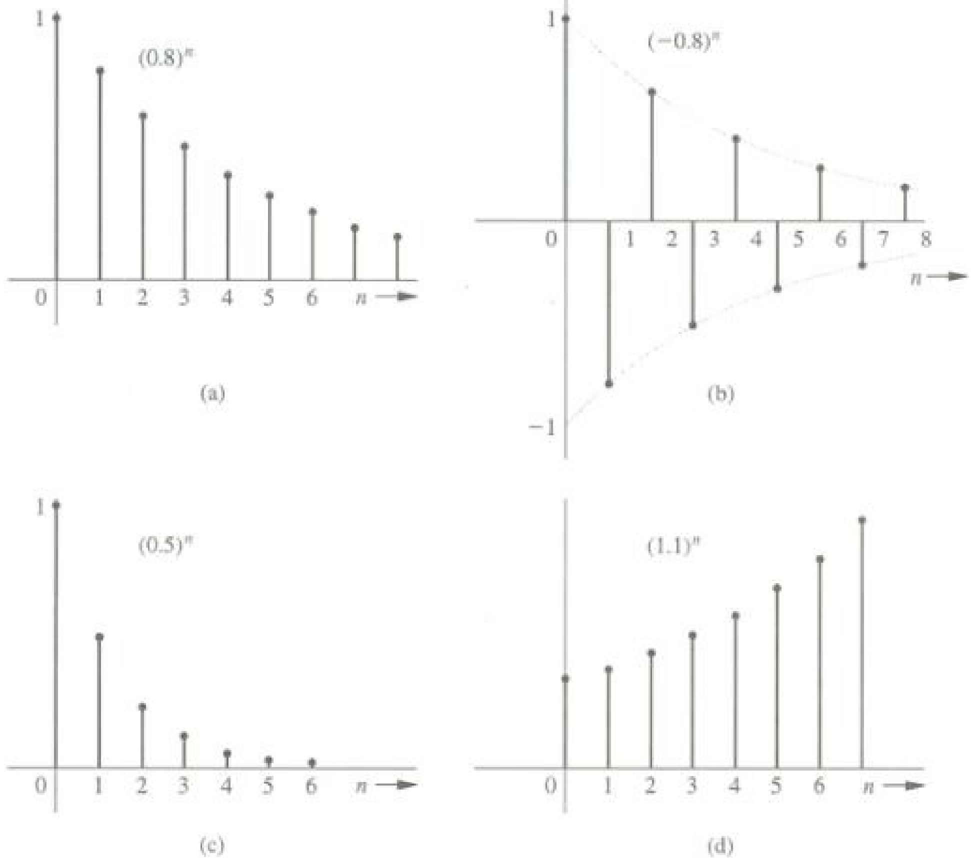


Figure 3.9 Discrete-time exponentials γ^n.

3.3. Some Useful DT Signal Models

3.3-4 DT Sinusoid cos(Ωn+Θ)

- DT Sinusoid : $c\cos(\Omega n+\theta)=c\cos(2\pi Fn+\theta)$
  - c : Amplitude
  - θ : phase in radians
  - Ω_n : angle in radians
  - Ω : radians per sample
  - F : DT freq. (radians/2π) per sample or cycles for sample

$$F=\frac{1}{N_0},\qquad N_0 \text{ : period (samples/cycle)}$$

3.3. Some Useful DT Signal Models

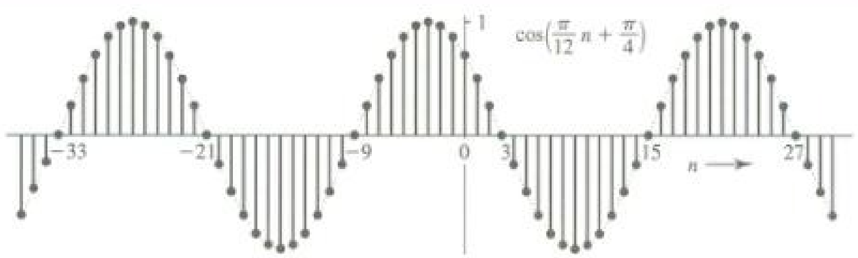


Figure 3.10 A discrete-time sinusoid $\cos\left(\frac{\pi}{12}n + \frac{\pi}{4}\right)$.

- About Figure 3.10

$$\cos\left(\frac{\pi}{12}n+\frac{\pi}{4}\right) \Rightarrow \Omega=\frac{\pi}{12} \Rightarrow F=\frac{\Omega}{2\pi}=\frac{1}{24} \quad \text{(cycles/sample)}$$

- $\cos(-\Omega n+\theta)=\cos(\Omega n-\theta),$ same freq. $|\Omega|$

➢ Sampled CT Sinusoid Yields a DT Sinusoid

- CT sinusoid

$$\cos wt = \underset{T}{} \ \underset{Seconds}{\overset{sampling}{\rightarrow}} \ x[n]=\cos wnT=\cos\Omega n, \qquad \Omega=wT$$

3.3. Some Useful DT Signal Models

3.3-5 DT Complex Exponential $e^{j\Omega n}$

$$e^{j\Omega n} = \cos\Omega n + j\sin\Omega n$$

$$e^{-j\Omega n} = \cos\Omega n - j\sin\Omega n$$

freq : $|\Omega|$

$$e^{j\Omega n} = re^{j\theta}$$

$$r = 1, \quad \theta = n\Omega$$

a point on a unit circle at an angle of $n\Omega$

3.4. Examples of DT Systems

3.4-0

- Example 3.4 (Savings Account)
  - input $x[n]$: a deposit in a bank regularly at an interval T.
  - output $y[n]$: balance
  - interest r : per dollar per period T.

  balance y[n] = previous balance y[n-1] + interest on y[n-1]
  + deposit x[n]

  $$\therefore y[n] = y[n-1] + ry[n-1] + x[n] = (1+r)\,y[n-1] + x[n]$$ or

  $$y[n] - ay[n-1] = x[n], \quad a = 1+r \quad (3.9a)$$

  delay operator form (causal)

  - withdrawal : negative deposit, $-x[n]$
  - loan payment problem : $y[0] = -M$ or $-x[0] = -M$
  - $y[n+1] - ay[n] = x[n+1]: \quad n \rightarrow n+1$ (3.9b)

    advance operator form (noncausal)

3.4. Examples of DT Systems

- Block diagram

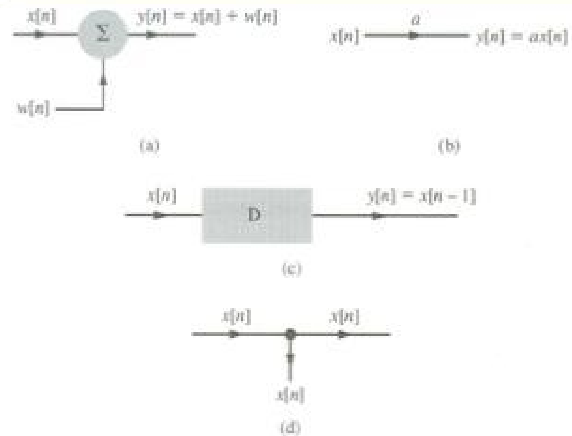


Figure 3.11 Schematic representations of basic operations on sequences.

(a)addition, (b)scalar multiplication, (c)delay, (d)pickoff node

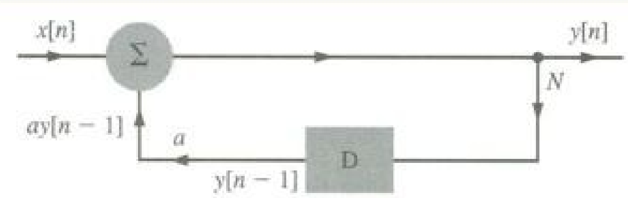


Figure 3.12 Realization of the savings account system.

3.4. Examples of DT Systems

- Example 3.5 (Scalar Estimate)

n : n th semester
$x[n]$: students enrolled in a course requiring a certain textbook
$y[n]$: new copies of the book sold in the n th semester.

- ¼ of students resell the text at the end of the semester.
- book life is three semesters.

$$y[n]+\frac{1}{4}y[n-1]+\frac{1}{16}y[n-2]=x[n] \qquad (3.10a)$$

$$y[n+2]+\frac{1}{4}y[n+1]+\frac{1}{16}y[n]=x[n+2] \qquad (3.10b)$$

- Block diagram

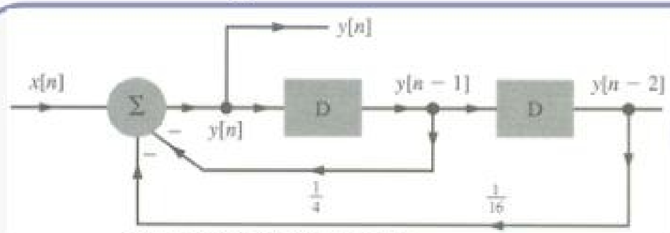


Figure 3.13 Realization of the system representing sales estimate in Example 3.5.

$$y[n]=-\frac{1}{4}y[n-1]-\frac{1}{16}y[n-2]+x[n] \qquad (3.10c)$$

3.4. Examples of DT Systems

- Example 3.6 (1) (Digital Differentiator)
  Design a DT system to differentiate CT signals. Signal bandwidth is below 20kHz (audio system)

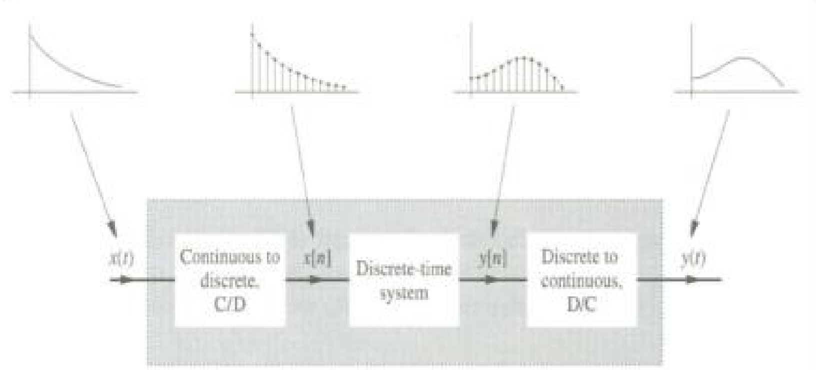


Figure 3.2 Processing a continuous-time signal by means of a discrete-time system.

$$x[n] = x(nT), \quad y[n] = y(nT)$$

$$y(t) = \frac{dx}{dt}, \quad t = nT$$

$$y(nT) = \left.\frac{dx}{dt}\right|_{t=nT} = \lim_{T\to 0}\frac{1}{T}\left[x(nT) - x((n-1)T)\right]$$: backward difference

3.4. Examples of DT Systems

- Example 3.6 (2)

$$\therefore\ y[n] = \lim_{T \to 0} \frac{1}{T}\{x[n] - x[n-1]\} \cong \frac{1}{T}\{x[n] - x[n-1]\}$$

T : Sufficiently small

$$T \le \frac{1}{2 \times highest \quad freq.} = \frac{1}{40{,}000} = 25\mu s$$

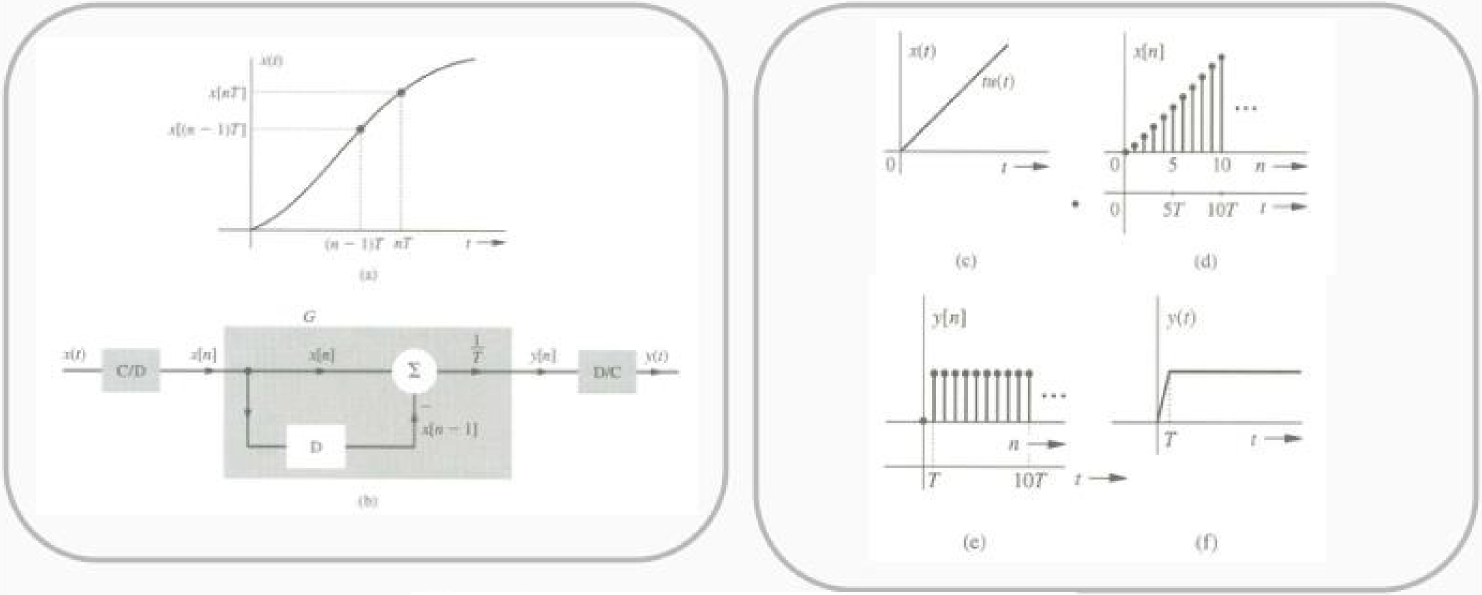


Figure 3.14 Digital differentiator and its realization.

3.4. Examples of DT Systems

- Example 3.6 (3)

  < in case of $x(t)=t$ >

  $$x[n]=x(t)\big|_{t=nT}=t\big|_{t=nT}=nT \quad t\geq 0$$

  $$y[n]=\frac{1}{T}\{nT-(n-1)T\}=1 \quad n\geq 1$$

  - Forward difference

  $$y[n]=\frac{1}{T}\{x[n+1]-x[n]\}$$

3.4. Examples of DT Systems

➢ Example 3.7 (Digital Integrator)
Design a digital integrator as in example 3.6

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau, \quad t = nT$$

$$y(nT) = \lim_{T\to 0} \sum_{k=-\infty}^{\infty} x(kT)T$$

$$y[n] = \lim_{T\to 0} T \sum_{k=-\infty}^{\infty} x[k], \quad T \to 0$$

$$\cong T \sum_{k=-\infty}^{n} x[k] \quad : \quad \text{accumulator}$$

$$\therefore\ y[n] - y[n-1] = Tx[n]$$

block diagram : similar to that of Figure 3.12 ($a = 1$)

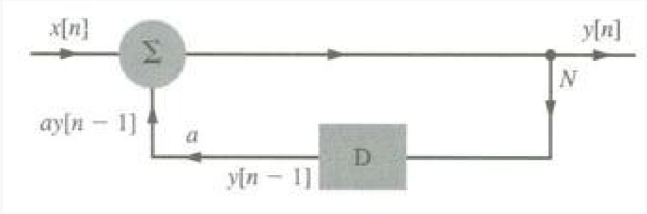


Figure 3.12 Realization of the savings account system.

3.4. Examples of DT Systems

- Recursive & Non-recursive Forms of Difference Equation

$$y[n] = T\sum_{k=-\infty}^{\infty} x[k] \quad (3.14a), \qquad y[n] - y[n-1] = Tx[n] \quad (3.14b)$$

- Kinship of Difference Equations to Differential Equations
  - Differential eq. can be approximated by a difference eq. of the same order.
  - The approximation can be made as close to the exact answer as possible by choosing sufficiently small value for T.

- Order of a Difference Equation
  - The highest – order difference of the output or input signal, whichever is higher.
- Analog, Digital, CT & DT Systems
  - DT, CT $\longrightarrow$ the nature of horizontal axis

    Analog, Digital $\longrightarrow$ the nature of vertical axis.

  -DT System : Digital Filter

  -CT System : Analog Filter

  -C/D, D/C : A/D, D/A

3.4. Examples of DT Systems

- Advantage of DSP
  1. Less sensitive to change in the component parameter values.
  2. Do not require any factory adjustment , easily duplicated in volume, single chip (VLSI)
  3. Flexible by changing the program
  4. A greater variety of filters
  5. Easy and inexperience storage without deterioration
  6. Extremely low error rates, high fidelity and privacy in coding
  7. Serve a number of inputs simultaneously by time-sharing, easier and efficient to multiplex several D. signals on the same channel
  8. Reproduction with extreme reliability

- Disadvantage of DSP
  1. Increased system complexity (A/D, D/A interface)
  2. Limited range of freq. available in practice (about tens of MHz)
  3. Power consumption

3.4. Examples of DT Systems

3.4-1 Classification of DT Systems

- Linearity & Time Invariance
  - $k_1 x_1 + k_2 x_2 \rightarrow k_1 y_1 + k_2 y_2$
  - Systems whose parameters do not change with time (n)
  - If the input is delayed by K samples, the output is the same as before but delayed by K samples.

    Ex) $y[n] = e^{-n} x[n]$ for $x_1[n]$ and $x_2[n] = x_1[n - N_0]$
- Causal & Noncausal Systems
  - output at any instant $n = k$ depends only on the value of the input $x[n]$ for $n \leq k$
- Invertible & Noninvertible Systems
  - S is invertible if an inverse system S_i exists s.t. the cascade of S and S_i results in an identity system

    Ex) unit delay $\longleftrightarrow$ unit advance (noncausal)

    Cf) $y[n] = x[Mn]$, $y[n] = \cos x[n]$, $y[n] = |x[n]|$

3.4. Examples of DT Systems

- Stable & Unstable Systems (1)
  - The condition of BIBO (Boundary Input Boundary Output) and external stability

$$y[n] = h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

$$|y[n]| = \left|\sum_{m=-\infty}^{\infty} h[m]x[n-m]\right| \le \sum_{m=-\infty}^{\infty} |h[m]||x[n-m]|$$

If $x[n]$ is bounded, then $|x[n-m] < K_1 < \infty|$, and

$$|y[n]| \le K_1 \sum_{m=-\infty}^{\infty} |h[m]|$$

Clearly the output is bounded if the summation on the right-hand side is bounded; that is

$$\sum_{n=-\infty}^{\infty} |h[n]| < K_2 < \infty$$

3.4. Examples of DT Systems

- Stable & Unstable Systems (2)
  - Internal (Asymptotic) Stability

For LTID systems,

$$\gamma = |\gamma| e^{j\beta} \quad \text{and} \quad \gamma^n = |\gamma|^n e^{j\beta n}$$

Since the magnitude of $e^{j\beta n}$ is 1, it is not necessary to be considered.
Therefore in case of $|\gamma|^n$

if	$\|\gamma\| < 1, \quad \gamma^n \to 0$	as	$n \to \infty$	(stable)
if	$\|\gamma\| > 1, \quad \gamma^n \to \infty$	as	$n \to \infty$	(unstable)
if	$\|\gamma\| = 1, \quad \|\gamma\|^n = 1$	for all n		(unstable)

3.4. Examples of DT Systems

- Memoryless Systems & Systems with Memory
  - Memoryless : the response at any instant n depends at most on the input at the same instant n .

    Ex) $$y[n] = \sin x[n]$$

  - With memory : depends on the past, present and future values of the input.

    Ex) $$y[n] - y[n-1] = x[n]$$

3.5. DT System Equations

- Difference Equations

$$y[n+N]+a_1 y[n+N-1]+\cdots+a_{N-1}y[n+1]+a_N y[n]=b_{N-M}x[n+M]$$
$$+b_{N-M+1}x[n+M-1]+\cdots+b_{N-1}x[n+1]+b_N x[n] \quad (3.16)$$

order : $\max(N,M)$

- Causality Condition
  - causality : $M \le N$
    if not, $y[n+N]$ would depend on $x[n+M]$
  - if $M=N$,

$$y[n+N]+a_1 y[n+N-1]+\cdots+a_{N-1}y[n+1]+a_N y[n]=b_0 x[n+N]$$
$$+b_1 x[n+N-1]+\cdots+b_{N-1}x[n+1]+b_N x[n] \quad (3.17a)$$

  - delay operator form

$$y[n]+a_1 y[n-1]+\cdots+a_{N-1}y[n-N+1]+a_N y[n-N]=b_0 x[n]$$
$$+b_1 x[n-1]+\cdots+b_{N-1}x[n-N+1]+b_N x[n-N] \quad (3.17b)$$

3.5. DT System Equations

3.5-1 Recursive (Iterative) Solution of Difference Equations

- $y[n] = -a_1 y[n-1] - a_2 y[n-2] - \cdots - a_N y[n-N] + b_0 x[n]$
  $+ b_1 x[n-1] + \cdots + b_{N-1} x[n-N+1] + b_N x[n-N]$ (3.17c)

  $y[0]$:
  - N initial conditions
  - input $x[0]$
  - $x[-n] = 0$ for causality

- Example 3.8 (1)

Solve iteratively

$$y[n] - 0.5y[n-1] = x[n] \tag{3.18a}$$

with initial condition $y[-1] = 16$ and causal input $x[n] = n^2$ (starting at $n = 0$). This equation can be expressed as

$$y[n] = 0.5y[n-1] + x[n] \tag{3.18b}$$

If we set $n = 0$ in this equation, we obtain

$$\begin{aligned} y[0] &= 0.5y[-1] + x[0] \\ &= 0.5(16) + 0 = 8 \end{aligned}$$

3.5. DT System Equations

- Example 3.8 (2)

Now, setting $n = 1$ in Eq. (3.18b) and using the value $y[0] = 8$ (computed in the first step) and $x[1] = (1)^2 = 1$, we obtain

$$y[1] = 0.5(8) + (1)^2 = 5$$

Next, setting $n = 2$ in Eq. (3.18b) and using the value $y[1] = 5$ (computed in the previous step) and $x[2] = (2)^2$, we obtain

$$y[2] = 0.5(5) + (2)^2 = 6.5$$

Continuing in this way iteratively, we obtain

$$y[3] = 0.5(6.5) + (3)^2 = 12.25$$

$$y[4] = 0.5(12.25) + (4)^2 = 22.125$$

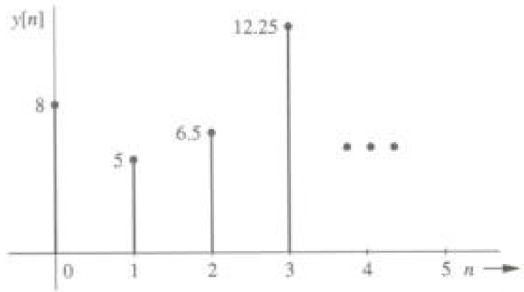


Figure 3.15 Iterative solution of a difference equation.

3.5. DT System Equations

- Example 3.9

Solve iteratively

$$y[n+2] - y[n+1] + 0.24y[n] = x[n+2] - 2x[n+1] \quad (3.19)$$

with initial conditions $y[-1] = 2$, $y[-2] = 1$ and a causal input $x[n] = n$ (starting at $n = 0$). The system equation can be expressed as

$$y[n+2] = y[n+1] - 0.24y[n] + x[n+2] - 2x[n+1] \quad (3.20)$$

Setting $n = -2$ and then substituting $y[-1] = 2$, $y[-2] = 1$, $x[0] = x[-1] = 0$ (recall that $x[n] = n$ starting at $n = 0$), we obtain

$$y[0] = 2 - 0.24(1) + 0 - 0 = 1.76$$

Setting $n = -1$ in Eq. (3.20) and then substituting $y[0] = 1.76$, $y[-1] = 2$, $x[1] = 1$, $x[0] = 0$, we obtain

$$y[1] = 1.76 - 0.24(2) + 1 - 0 = 2.28$$

Setting $n = 0$ in Eq. (3.20) and then substituting $y[0] = 1.76$, $y[1] = 2.28$, $x[2] = 2$ and $x[1] = 1$ yields

$$y[2] = 2.28 - 0.24(1.76) + 2 - 2(1) = 1.8576$$

- Closed – form solution

3.5. DT System Equations

- Operation Notation
  - Differential eq. ⟶ Operator D for differentiation
  - Difference eq. ⟶ Operator E for advancing a sequence.
    - $Ex[n] \equiv x[n+1], \quad E^2x[n] \equiv x[n+2]$
    - $y[n+1] - ay[n] = x[n+1] \quad \rightarrow \quad Ey[n] - ay[n] = Ex[n]$

      $(E-a)y[n] = Ex[n]$
    - $y[n+2] + \frac{1}{4}y[n+1] + \frac{1}{16}y[n] = x[n+2] \rightarrow$

      $(E^2 + \frac{1}{4}E + \frac{1}{16})y[n] = E^2x[n]$

$$(E^N + a_1E^{N-1} + \cdots + a_{N-1}E + a_N)y[n] = (b_0E^N + b_1E^{N-1} + \cdots + b_{N-1}E + b_N)x[n] \quad (3.24a)$$

$$Q[E]y[n] = P[E]x[n] \quad (3.24b)$$

$$Q[E] = E^N + a_1E^{N-1} + \cdots + a_{N-1}E + a_N \quad (3.24c)$$

$$P[E] = b_0E^N + b_1E^{N-1} + \cdots + b_{N-1}E + b_N \quad (3.24d)$$

3.5. DT System Equations

- Response of Linear DT Systems

$$(E^N + a_1 E^{N-1} + \cdots + a_{N-1}E + a_N)y[n]$$
$$= (b_0 E^N + b_1 E^{N-1} + \cdots + b_{N-1}E + b_N)x[n] \qquad (3.24a)$$

$$Q[E]y[n] = P[E]x[n] \qquad (3.24b)$$

  - LTID system
  - General solution = ZIR (zero input response)
    + ZSR (zero state response)

END