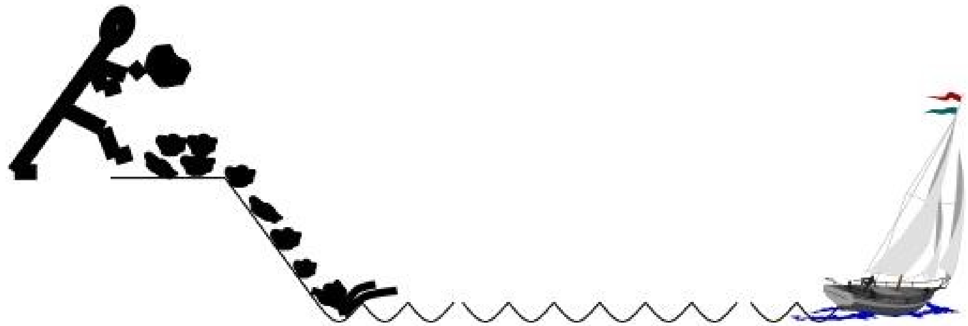
Basics of Signal Processing



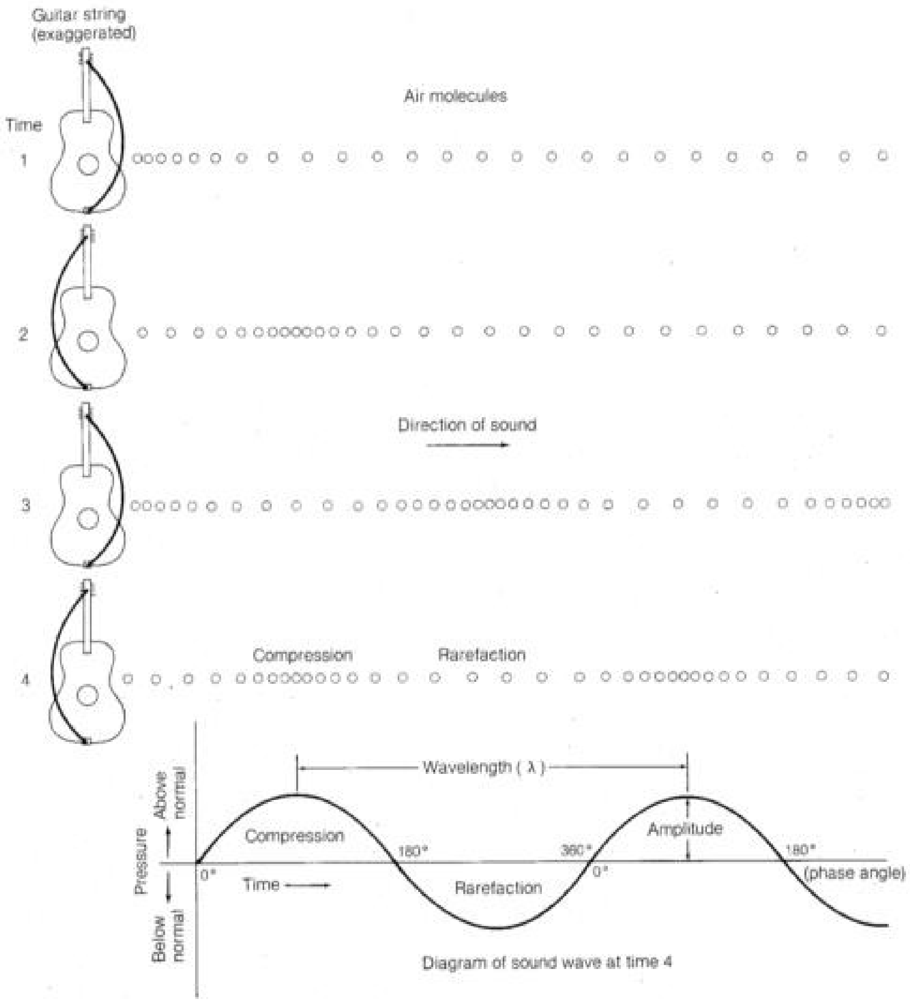
ACTION

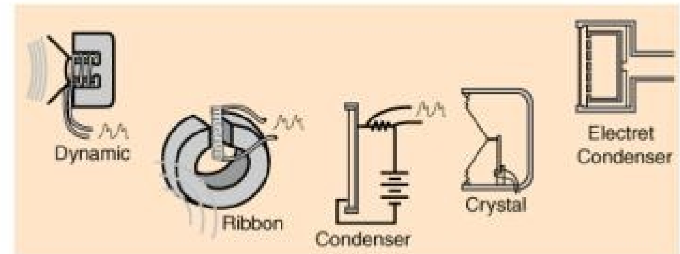
describe waves in terms of their significant features

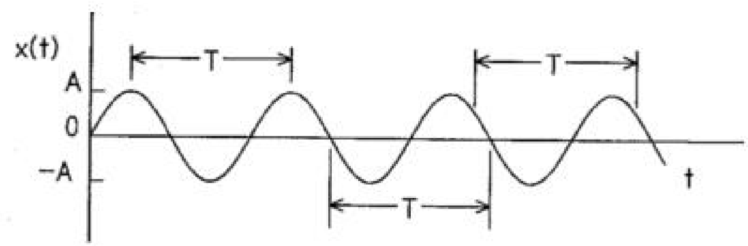
understand the way the waves originate

effect of the waves

will the people in the boat notice ?





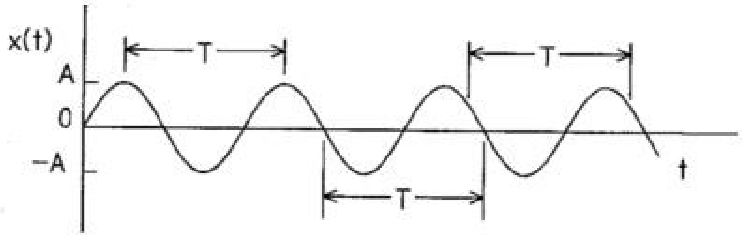


frequency = 1/T

sine wave

- period (frequency)
- amplitude
- phase

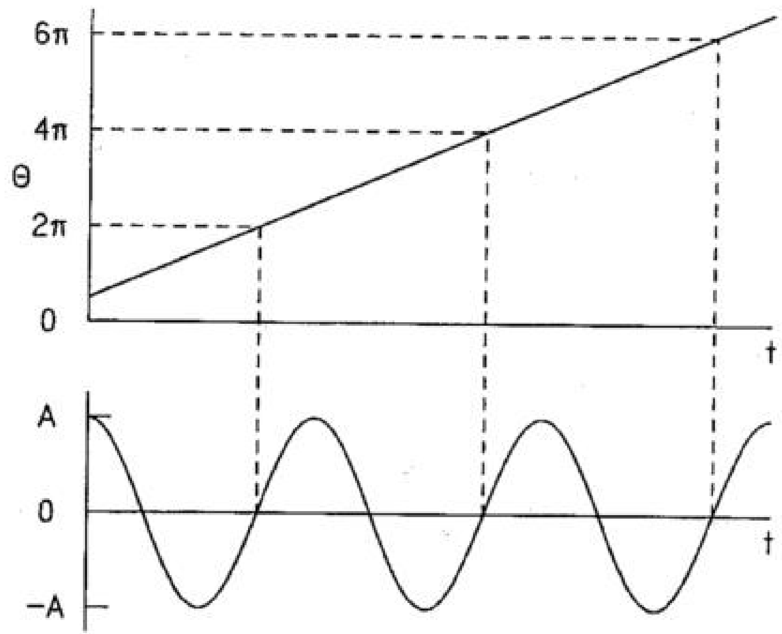
λ = speed of sound $\times$ T, where T is a period

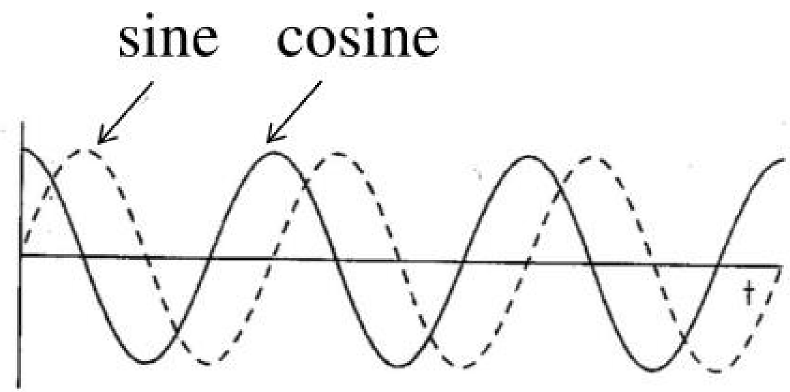


$$f(t) = A\sin(2\pi t / T + \phi)$$

$$f(t) = A\sin(\omega t + \phi)$$

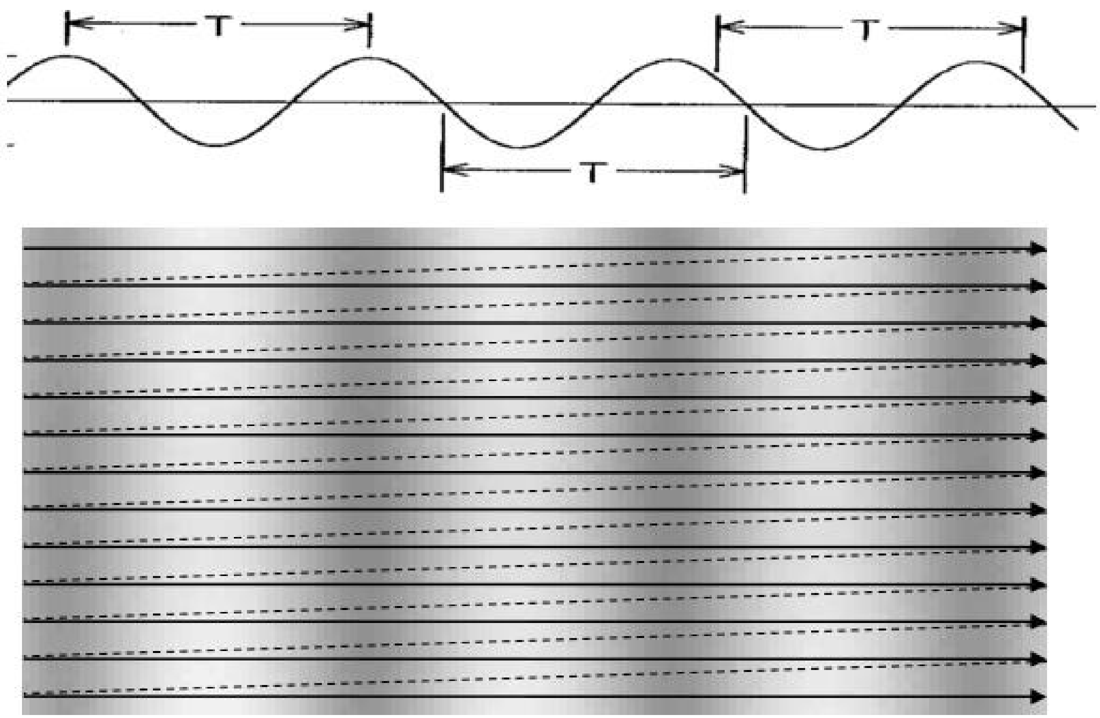
Phase Φ

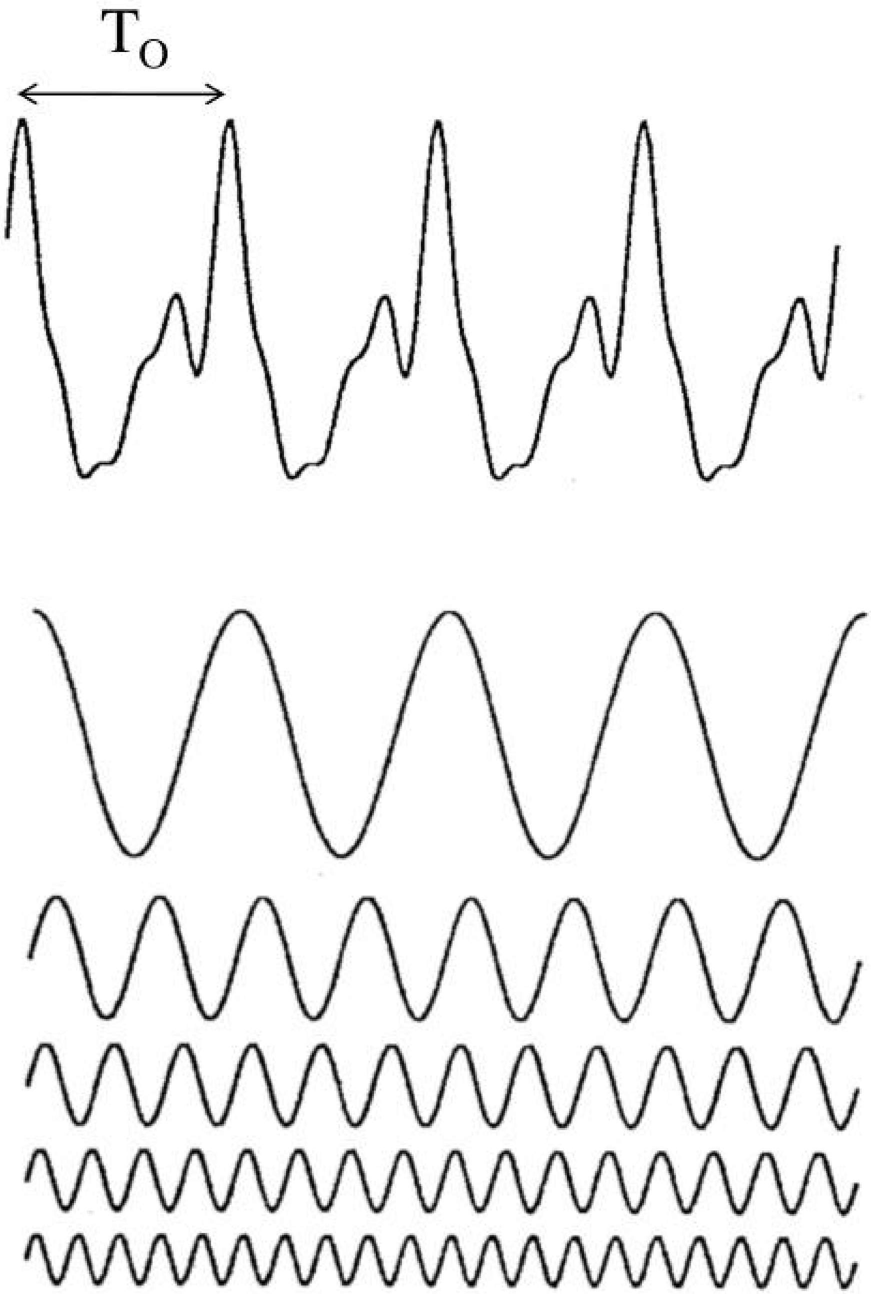




$$A\sin(\omega t + \pi/2)$$
$$= A\cos(\omega t)$$

Sinusoidal grating of image





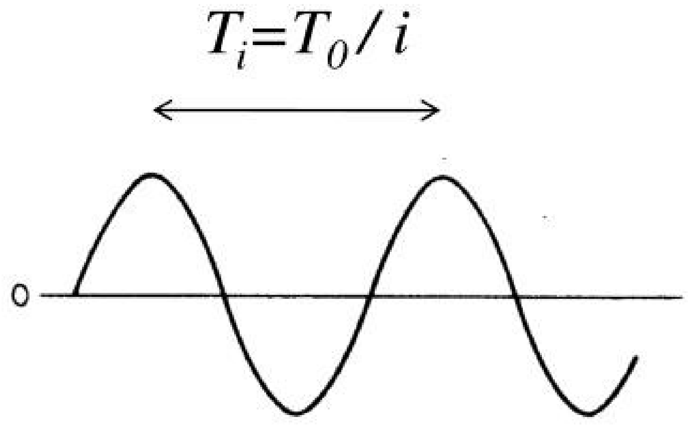
- Fourier idea
  - describe the signal by a sum of other well defined signals

Fourier Series

A periodic function as an infinite weighted sum of simpler periodic functions!

$$f(t) = \sum_{i=0}^{\infty} w_i f_i(t)$$

A good simple function



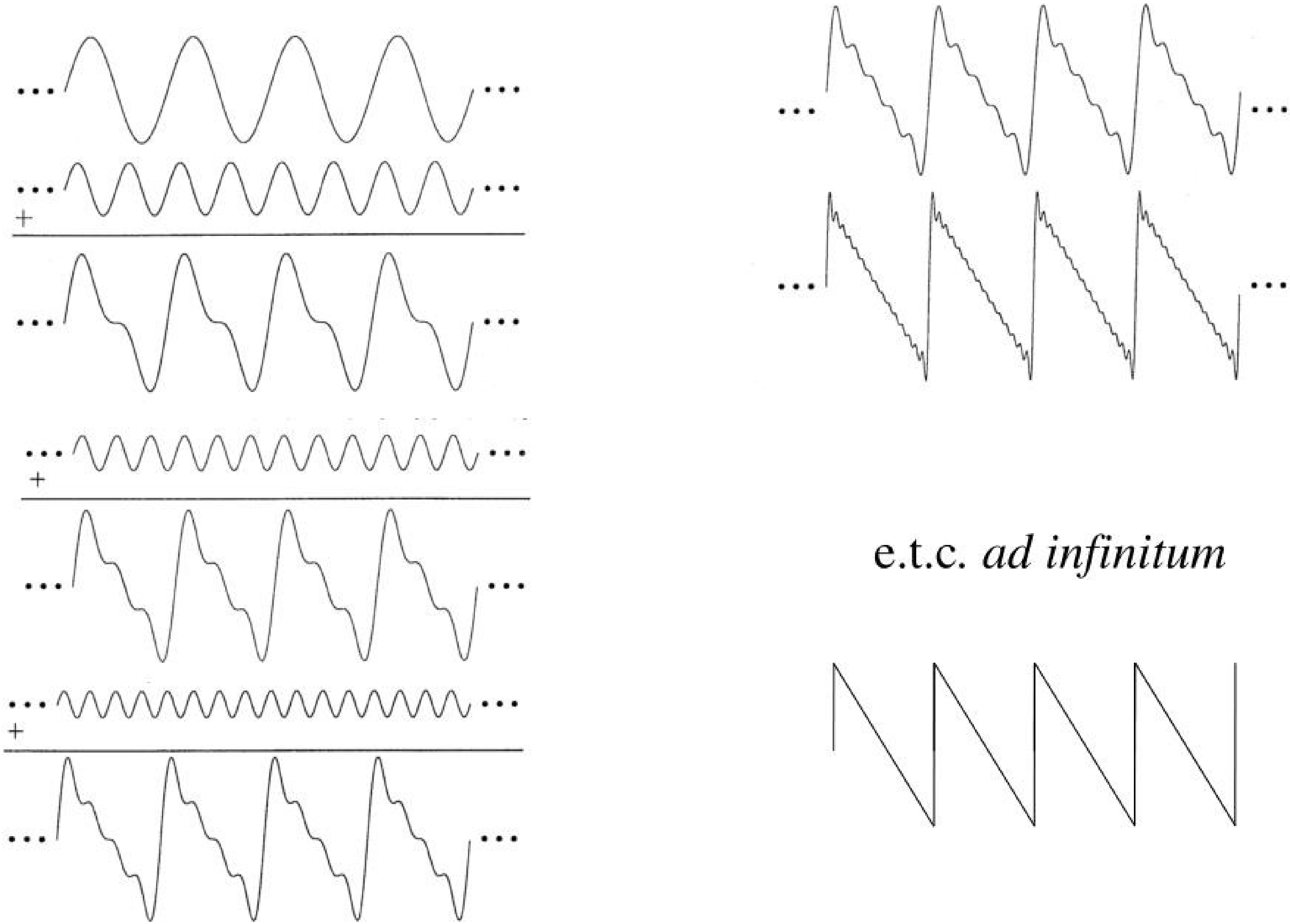
$$f_i(t) = sin(i\omega_0 t + \phi),$$

$$\text{where } \omega_0 = 2\pi / T_0$$

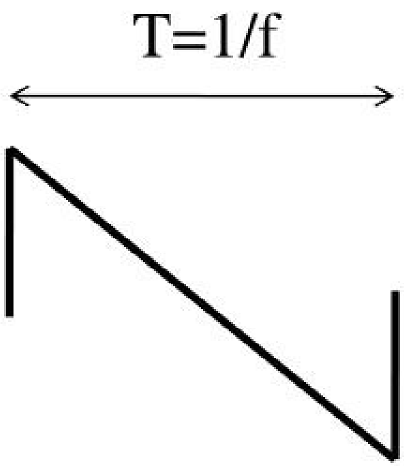
$$f(t) = \sum_{i=1}^{\infty} k_i \sin(i\omega_0 + \varphi_n)$$

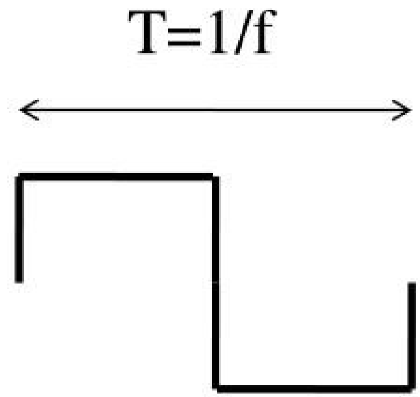
$$= \sum_{i=1}^{\infty} [b_i \sin(i\omega_0) + a_i \cos(i\omega_0)]$$

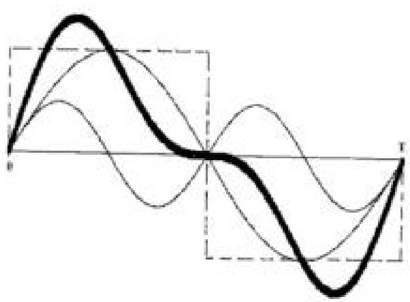
$$= \operatorname{Re} \sum_{i=0}^{\infty} \hat{c}_i \cdot e^{-j\omega_0 n}, \hat{c} - complex$$

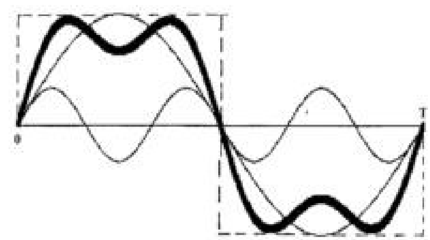

e.t.c. *ad infinitum*

$$f(t) = \sum_{i=1}^{\infty} k_i \sin(i\omega_0 + \varphi_n) = \sum_{i=1}^{\infty} [b_i \sin(i\omega_0) + a_i \cos(i\omega_0)]$$





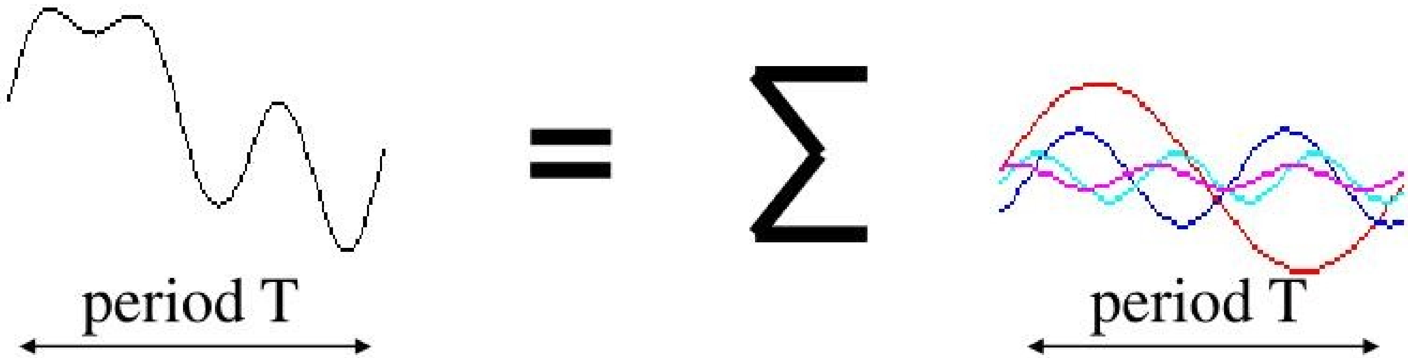




e.t.c……

e.t.c……

Fourier's Idea



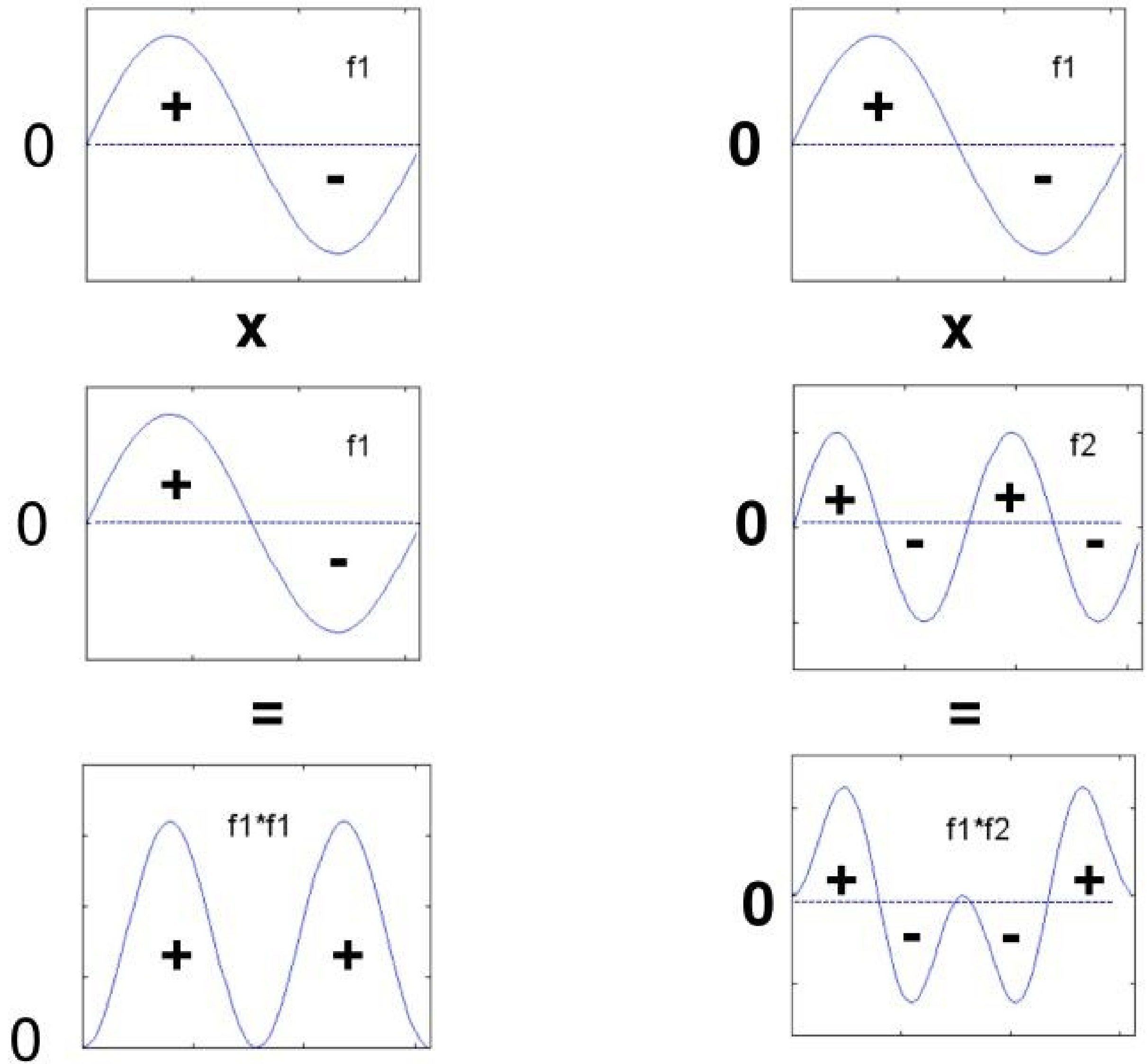
Describe complicated function as a weighted sum of simpler functions!

-simpler functions are known

-weights can be found

Simpler functions - sines and cosines are orthogonal on period T, i.e.

$$\int_0^T f(mt)\cdot f(nt)\,dt = 0 \text{ for } m \neq n$$



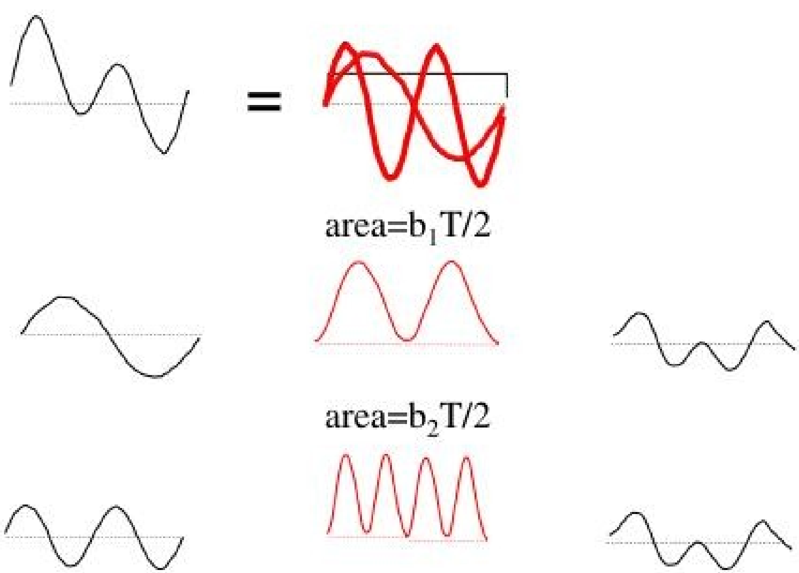
area is positive (T/2)

area is zero

$$f(t) = DC + \sum_{i=1}^{\infty}\left[a_i \cos(\frac{2\pi it}{T}) + b_1 \sin(\frac{2\pi it}{T})\right] = DC + a_1 \cos(\frac{2\pi t}{T}) + b_1 \sin(\frac{2\pi t}{T}) + a_2 \cos(\frac{4\pi t}{T}) + b_2 \sin(\frac{4\pi t}{T}) + a_3 \cos(\frac{6\pi t}{T}) + b_3 \sin(\frac{6\pi t}{T}) + ...$$

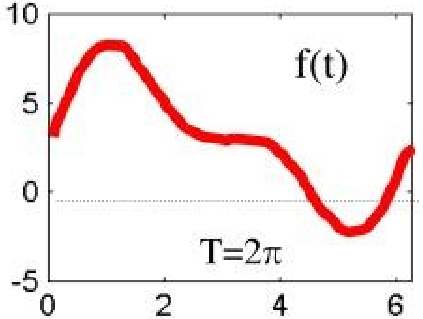
$$\int_0^T f(t)\sin(\frac{2\pi t}{T})dt = \int_0^T \{DC \sin(\frac{2\pi t}{T}) + a_1 \cos(\frac{2\pi t}{T})\sin(\frac{2\pi t}{T}) + b_1 \sin(\frac{2\pi t}{T})\sin(\frac{2\pi t}{T}) + a_2 \cos(\frac{4\pi t}{T})\sin(\frac{2\pi t}{T}) + b_2 \sin(\frac{4\pi t}{T})\sin(\frac{2\pi t}{T}) + ...\}dt$$

$0 \qquad 0 \qquad b_1T/2 \qquad 0 \qquad 0$ ....



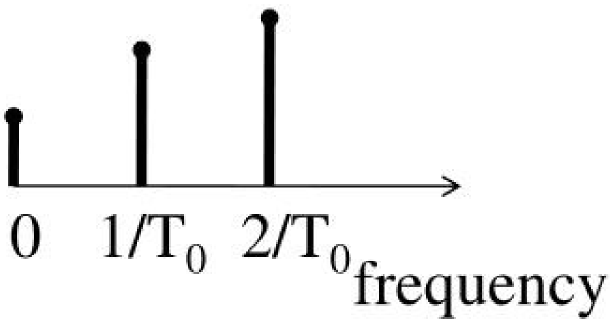
$$f(t) = DC + f_1(t) + f_2(t) = DC + b_1 \bullet \sin \omega t + b_2 \bullet \sin 2\omega t$$

area = DC

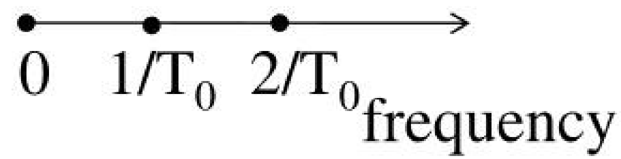


$$\int_0^T \sin^2(\frac{t}{T})\, dt = \frac{T}{2}$$

Magnitude spectrum



Phase spectrum

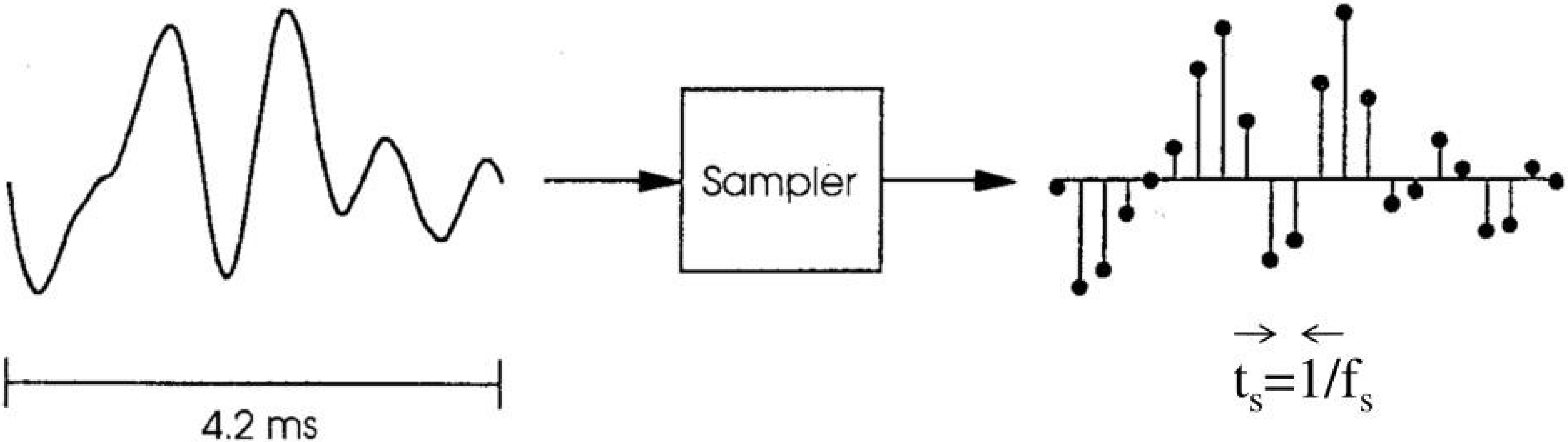


Spacing of spectral components is $f_0 = 1/T_0$

Aperiodic signal $T_0 \rightarrow \infty \Rightarrow$ frequency spacing $f_0 \rightarrow 0$

Discrete spectrum becomes **continuous** (Fourier integral)

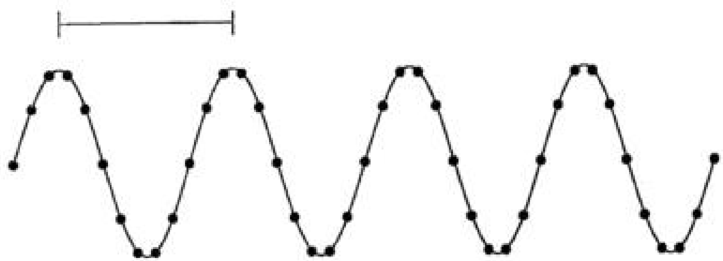
sampling



22 samples per 4.2 ms ⇔ 0.19 ms per sample ⇔ 5.26 kHz

Sampling

T = 10 ms (f = 1/T=100 Hz)



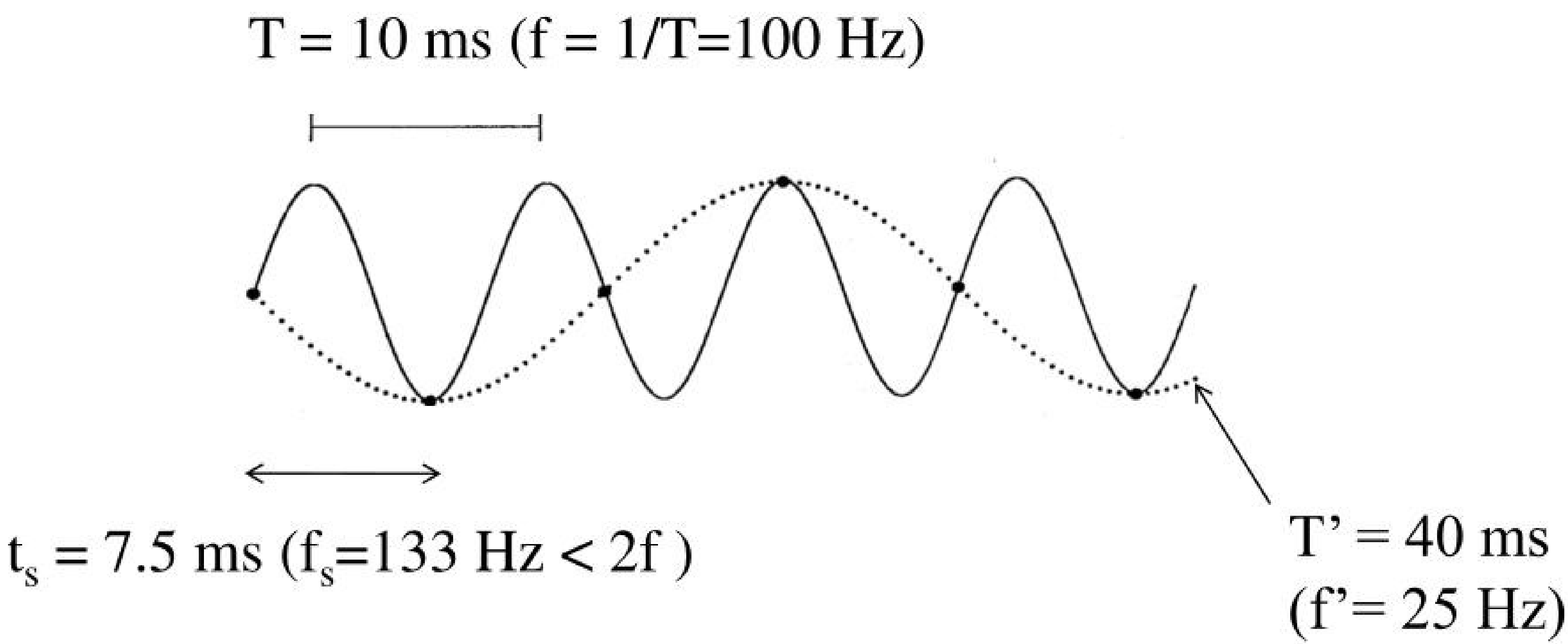
> 2 samples per period,
$f_s > 2 f$

Sinusoid is characterized by three parameters

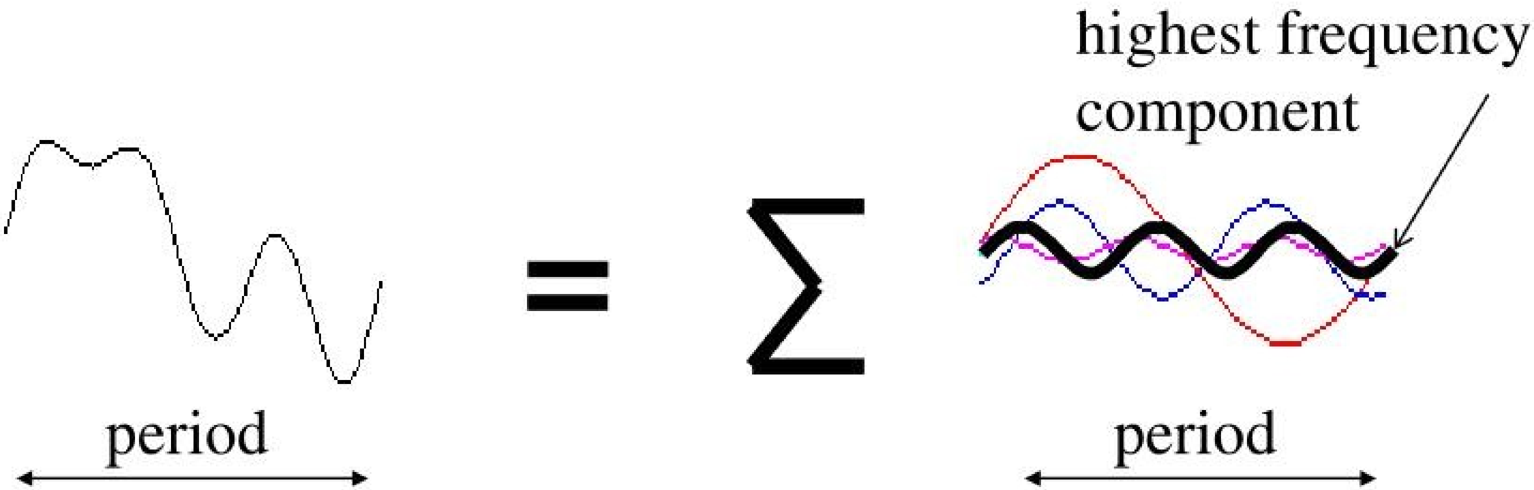
1. Amplitude
2. Frequency
3. Phase

We need at least three samples per the period

Undersampling



Sampling of more complex signals



Sampling must be at the frequency which is higher than the twice the highest frequency component in the signal !!!

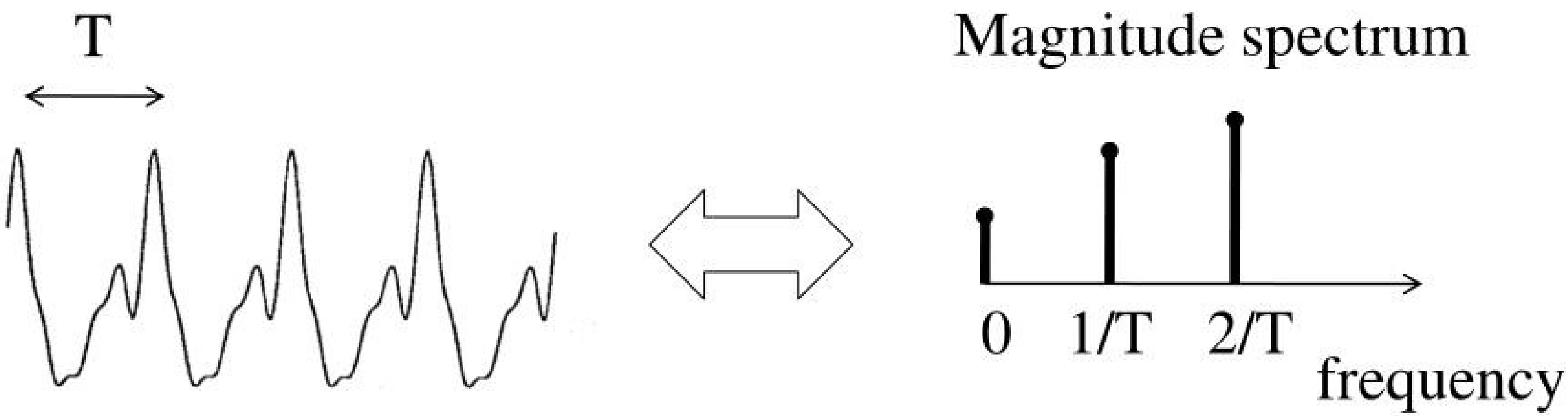
$$f_s > 2\ f_{max}$$

Sampling

1. Make sure you know what is the highest frequency in the signal spectrum f_{MAX}

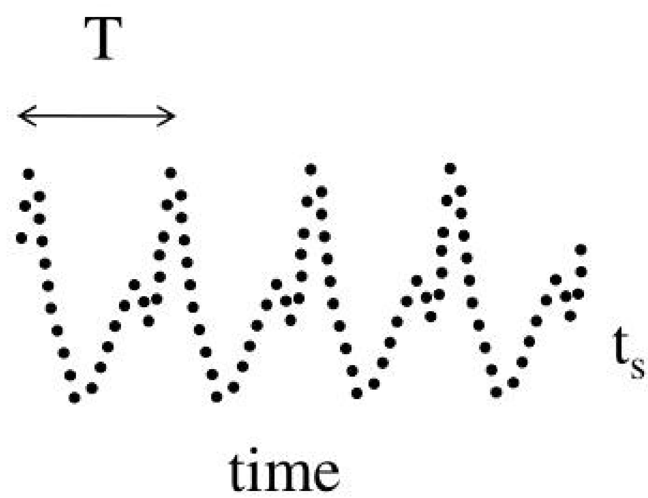
2. Chose sampling frequency $f_s > 2\ f_{MAX}$

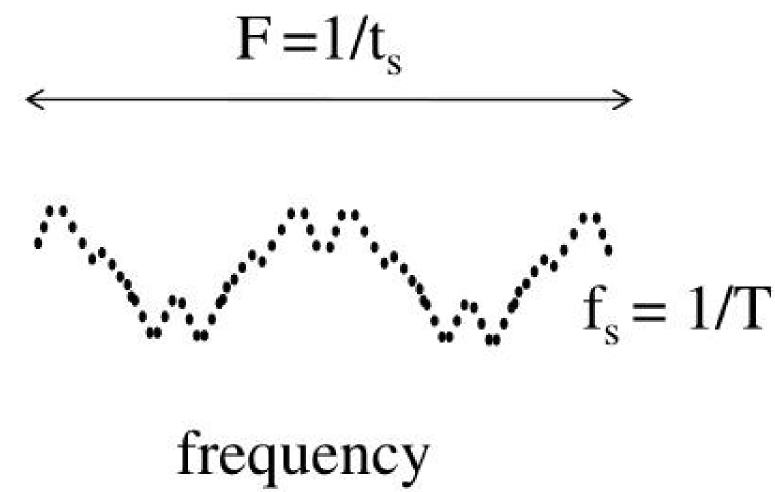
NO NEED TO SAMPLE ANY FASTER !



Periodicity in one domain implies discrete representation in the dual domain

Sampling in time implies periodicity in frequency !





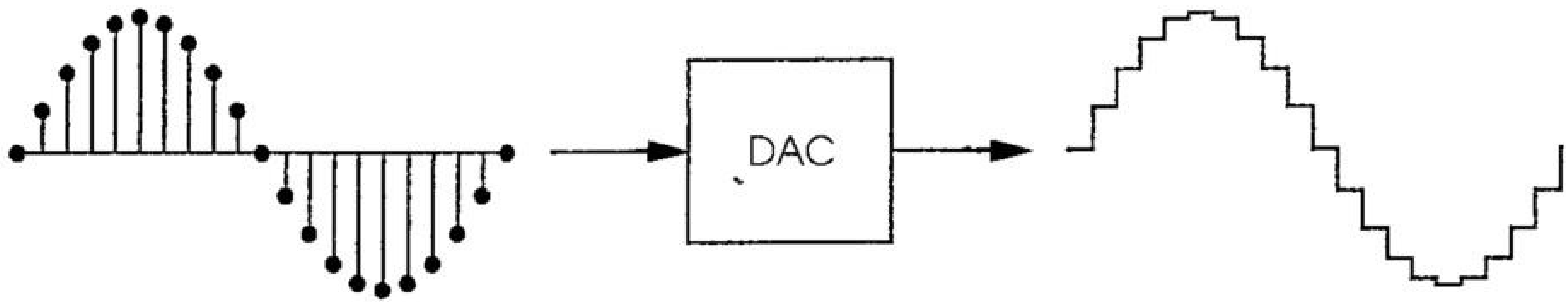
DISCRETE FOURIER TRANSFORM

$$x(n) = \tfrac{1}{N}\sum_{n=0}^{N-1} X(k)\cdot e^{j\frac{2\pi kn}{N}} \qquad X(k) = \tfrac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi kn}{N}}$$

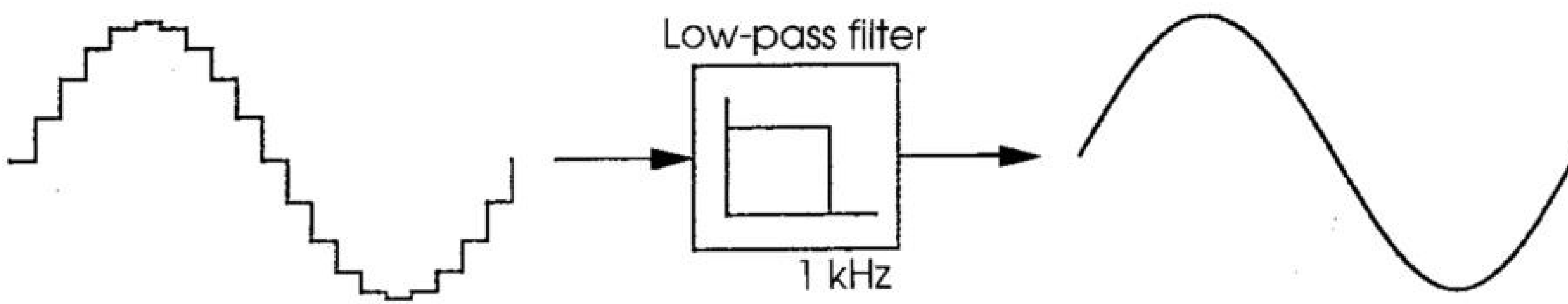
Discrete and periodic in both domains (time and frequency)

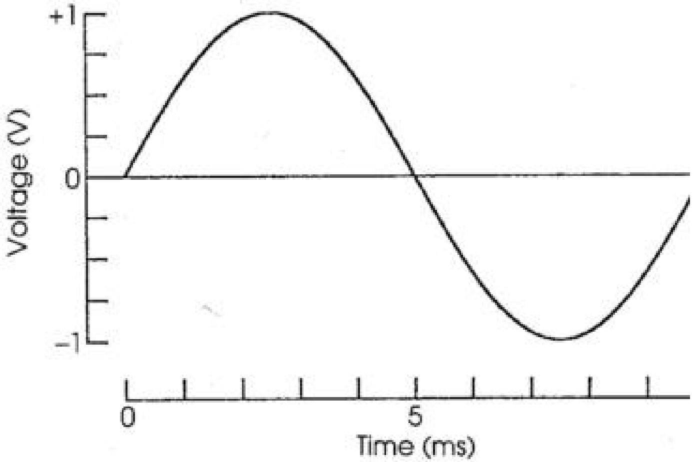
Recovery of analog signal

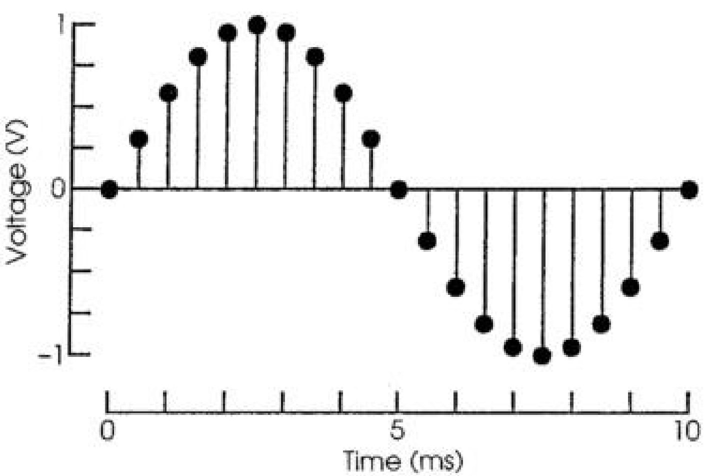
Digital-to-analog converter ("sample-and-hold")



Low-pass filtering

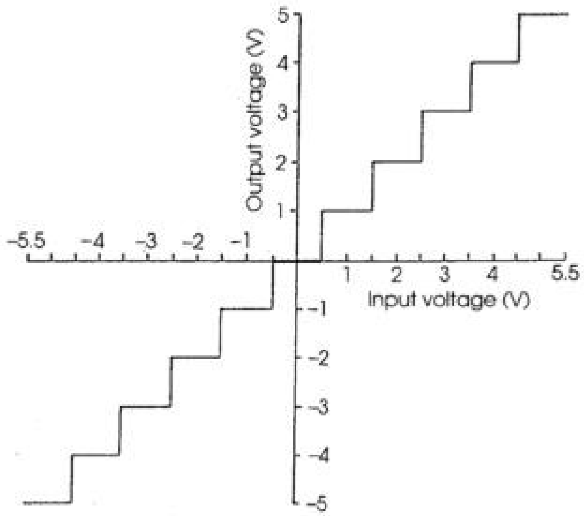




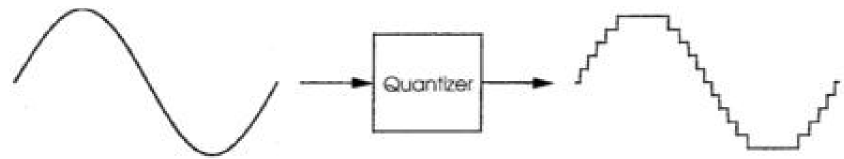


Time (ms)	
0.0	0.000000000000000
0.5	0.309016742003550
1.0	0.587784822932543
1.5	0.809016526452407
2.0	0.951056188292881
2.5	1.000000000000000
3.0	0.951057008296553
3.5	0.809018086192214
4.0	0.587786969730540
4.5	0.309019265716544
5.0	0.000000000000000
5.5	-0.309014218288380
6.0	-0.587782676130406
6.5	-0.809014966706903
7.0	-0.951055368282511
7.5	-1.000000000000000
8.0	-0.951057828293529
8.5	-0.809019645926324
9.0	-0.587789116524398
9.5	-0.309021789427363
10.0	-0.000000000000000

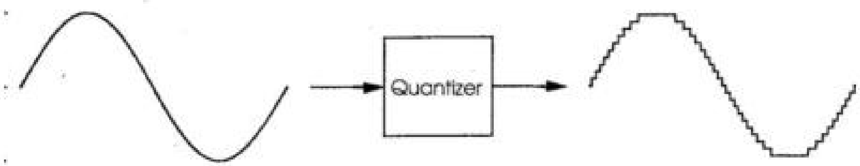
Quantization



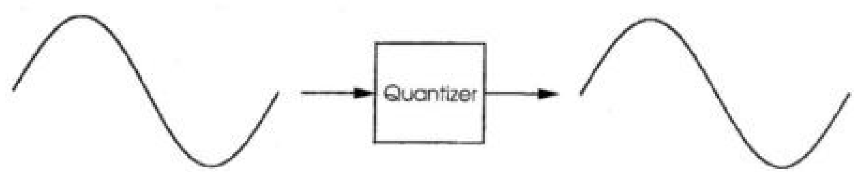
11 levels



21 levels

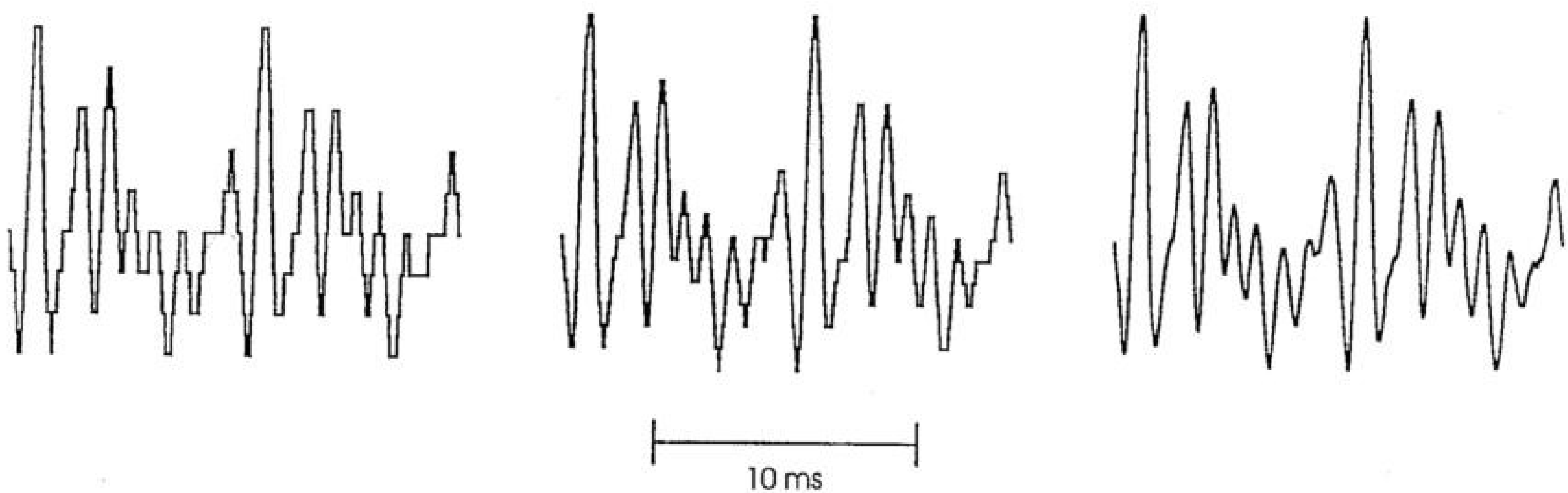


111 levels



a part of vowel /a/

16 levels (4 bits)   32 levels (5 bits)   4096 levels (12 bits)



Quantization

- Quantization error = difference between the real value of the analog signal at sampling instants and the value we preserve
- Less error ⇔ less "quantization distortion"